# International Workshop on the Multi-Phase Flow; Analysis, Modeling and Numerics

Waseda, November 28 – December 01, 2017

# Abstracts

TOP GLOBAL UNIVERSITY JAPAN

科研費 KAKENHI

早稲田大学 WASEDA University

## Bernardo Cockburn

School of Mathematics, University of Minnesota, Minnesota, USA


### New mixed and discontinuous Galerkin methods for the incompressible Navier-Stokes equations

We show how to systematically devise superconvergence mixed and hybridizable discontinuous Galerkin methods for the Navier-Stokes equations using meshes of arbitrary polyhedral elements. These new methods can be obtained by using as building blocks the corresponding methods for the steady-state diffusion problem. In the first talk, we show how to devise superconvergent mixed and hybridizable discontinuous Galerkin methods for steady-state diffusion by means of the recently obtained theory of M-decompositions. In the second talk, we show how to use these results to construct the corresponding methods for the Stokes system and then for the Navier-Stokes of incompressible fluid flow.

— — —


## Walter Craig

Department of Mathematics and Statistics, McMaster University, Ontario, Canada


### Four lectures on water waves

#### (1) Water waves and Hamiltonian partial differential equations

1. Physical derivation of the equations for free surface water waves
2. Derivation of the Zakharov Hamiltonian
3. Dirichlet – Neumann operator and its analysis

#### (2) Model equations for water waves

1. Canonical transformation theory
2. Shallow water scaling - the Kano – Nishida theorem
3. Boussinesq and KdV scaling limits
4. The nonlinear Schrödinger equation, and the modulational scaling limit

#### (3) Birkhoff normal forms

1. Gravity waves
2. Capillary - gravity waves
3. Formal normal forms in infinite depth

4. Analytic properties of normal forms transformations

**(4) Initial value problems**

1. Variational equations
2. Energy estimates
3. Nalimov's theorem, S. Wu's theorem
4. analytic properties of the solution map
5. Cases $x \in \mathbb{T}^{d-1}$ and $x \in \mathbb{R}^{d-1}$

— — —


## Toshiaki Hishida

Graduate School of Mathematics, Nagoya University, Nagoya, Japan


### Large time behavior of a generalized Oseen evolution operator, with applications to the Navier-Stokes flow past a rotating obstacle

Consider the motion of a viscous incompressible fluid in a 3D exterior domain when a rigid body moves with a prescribed time-dependent translational and angular velocities. For the linearized non-autonomous system, $L^q$-$L^r$ smoothing action near the initial time as well as generation of the evolution operator was shown by Hansel and Rhandi under reasonable conditions (J. Reine Angew. Math. 2014). In this presentation we develop the $L^q$-$L^r$ decay estimates of the evolution operator and then apply them to the Navier-Stokes initial value problem.

— — —


## Mads Kyed

Department of Mathematics, Technical University Darmstadt, Darmstadt, Germany


### Time-Periodic Navier-Stokes Equations

In this series of four lectures I will give an overview of the mathematical theory on time-periodic Navier-Stokes equations. A broad range of fluid flow problems involve some type of time-periodic forcing term that lead to Navier-Stokes models with time-periodic right-hand sides. Questions of existence, uniqueness, regularity and other properties of time-periodic solutions arise naturally. The mathematical investigation of the time-periodic problem, however, has not been anywhere as comprehensive as the investigation of the steady-state and initial-value Navier-Stokes equations. Classically, time-periodic solutions are studied in the realm of the initial-value problem, that is, as special solutions to the initial-value problem, but in many cases it can be an advantage to approach the time-periodic problem with the methodology one would employ for the steady-state case. The different techniques and associated challenges will be discussed. I will provide an overview of existing results and open problems. The four lectures are divided into the following topics:

1. Leray-Hopf type weak solutions, uniqueness, regularity.
2. Fundamental solution, Lp estimates, unbounded domains.
3. Asymptotic structure at spatial infinity, stability.
4. Generalizations to other equations.

— — —

## Tadahisa Funaki

Department of Mathematics, Waseda University, Tokyo, Japan


### Volume preserving mean curvature flow with stochastic term

In the workshop of last year, we discussed the sharp interface limit for a mass conserving Allen-Cahn equation with stochastic term and derived a stochastically perturbed volume preserving mean curvature flow in the limit. The local existence for the stochastic dynamics appearing in the limit was shown by rewriting it into an equation for the curvature, in a special situation that the limit interfaces are 2D convex curves. We extend our result in higher dimensional non-convex setting by rewriting the equation for signed distance functions. This is joint work with Satoshi Nakada and Satoshi Yokoyama.

— — —


## Matthias Hieber

Department of Mathematics, Technical University Darmstadt, Darmstadt, Germany


### Mild and Strong Periodic Solutions to Semilinear Evolution Equations

In this talk, we discuss various approaches to mild and strong periodic solutions to semilinear evolution equations and apply it to assorted examples ranging from incompressible fluid flow, stratified fluids and the primitive equations of ocean dynamics to models of electrophysiology. Our first approach is based on smoothing properties of the underlying linear equation and interpolation methods and yields results for small forces. A weak-strong uniqueness property allows us further to obtain strong periodic solutions in the case of the primitive equations, even for arbitrary large forces. Finally, we consider the bidomain operator and show how to obtain strong periodic solutions to the FitzHugh-Nagumo model.

— — —


## Hideo Kozono

Department of Mathematics, Waseda University, Tokyo, Japan


### Finite energy for the Navier-Stokes equations and Liouville-type theorems

Introducing a new notion of *generalized suitable weak solutions*, we prove validity of the energy inequality for such a class of weak solutions to the Navier-Stokes equations in the whole space $\mathbb{R}^n$. Although we need certain growth condition on the pressure, we may treat the class even with infinite energy quantity except for the initial velocity. As an application, Loiuville-type theorems are obtained.

This is the joint work with Prof. Yutaka Terasawa at Nagoya University and Prof. Yuta Wakasugi at Ehime University.

**References**

[1] Chae, D., *Remarks on the Liouville type problem in the stationary 3D Navier-Stokes equations.* arXiv:1502.04793v1.
[2] Giga, Y., Hsu, P.-Y., Maekawa, Y., *A Liouville theorem for the planer Navier-Stokes equations with the no-slip boundary condition and its application to a geometric regularity criterion*, Comm. Partial Differential Equations **39** (2014), 1906–1935.
[3] Koch, G., Nadirashvili, N.,Seregin, G., A.; Sverák, V. , *Liouville theorems for the Navier-Stokes equations and applications*, Acta Math. **203** (2009), 83–105.
[4] Kozono, H., Terasawa, Y., Wakasugi, Y., *A Remark on Liouville-type theorems for the stationary Navier-Stokes equations in three space dimensions*, J. Funct. Anal. **272** (2017), 804–818.

— — —

## Yasunori Maekawa

Department of Mathematics, Kyoto University, Kyoto, Japan

### On stationary two-dimensional flows around a fast rotating disk

We study the two-dimensional stationary Navier-Stokes equations describing flows around a rotating disk.

The existence of unique solutions is established for any rotating speed, and qualitative effects of a large rotation are described precisely by exhibiting a boundary layer structure and an axisymmetrization of the flow. This talk is based on the joint work with Isabelle Gallagher (IMJ-PRG, Université Paris-Diderot and DMA, Ecole Normale Supérieure de Paris) and Mitsuo Higaki (Kyoto University).

— — —

## Issei Oikawa

Department of Mathematics, Waseda University, Tokyo, Japan

### Superconvergence of the HDG method

It is known that the hybridizable discontinuous Galerkin (HDG) method is superconvergent in some cases, for example, when we use piecewise polynomials of degree $k$ to approximate all variables and triangular meshes. The superconvergence of the HDG method are well studied and various methods are proposed. They are classified into two categories. The first one is to use projected (or reduced) stabilization, which was firstly proposed by Lehrenfeld and Schöberl. The second one is to construct appropriate approximate spaces for the vector variable, in which the so-called $M$-decomposition is included. In this talk, we present a new superconvergent HDG method in the first category and some ideas to prove superconvergence properties in the second category.

— — —

Hirokazu Saito
Department of Mathematics, Waseda University, Tokyo, Japan

**Local solvability of the Navier-Stokes-Korteweg system on general domains**

In this talk, we would like to consider the Navier-Stokes-Korteweg system on general domains of $\mathbf{R}^N$, $N \geq 2$. Korteweg introduced in the early 1900s a stress tensor including the density gradients in order to model fluid capillarity effects. After that, Dunn and Serrin derived rigorously $\mathbf{K}(\rho) = (\kappa/2)(\Delta\rho^2 - |\nabla\rho|^2)\mathbf{I} - \kappa\nabla\rho\otimes\nabla\rho$ that is called a Korteweg tensor nowadays. In this approach, the momentum equation is turned into $\rho(\partial_t\mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u}) = \mathrm{Div}(\mathbf{S}(\mathbf{u}) + \mathbf{K}(\rho) - P(\rho)\mathbf{I})$ with the standard viscous stress $\mathbf{S}(\mathbf{u}) = \mu\mathbf{D}(\mathbf{u}) + (\nu - \mu)\mathrm{div}\mathbf{u}\mathbf{I}$ and the pressure $P(\rho)$, which is called the Navier-Stokes-Korteweg equation.

As the first step, we consider a resolvent problem arising from the Navier-Stokes-Korteweg system and prove the existence of $\mathcal{R}$-bounded solution operator families for the resolvent problem. The definition of the $\mathcal{R}$-boundedness is given by

**Definition 0.1.** *Let $X$, $Y$ be Banach spaces. A family of operators $\mathcal{T} \subset \mathcal{L}(X, Y)$ is called $\mathcal{R}$-bounded, if there exist $p \in [1, \infty)$ and a positive constant $C$ such that the following inequality holds true: For any $m \in \mathbf{N}$, $\{T_j\}_{j=1}^m \subset \mathcal{T}$, $\{x_j\}_{j=1}^m$ and for all sequences $\{r_j(u)\}_{j=1}^m$ of independent, symmetric, and $\{-1, 1\}$-valued random variables on $[0, 1]$,*

$$\left(\int_0^1 \Big\| \sum_{j=1}^m r_j(u) T_j x_j \Big\|_Y^p du\right)^{1/p} \leq C \left(\int_0^1 \Big\| \sum_{j=1}^m r_j(u) x_j \Big\|_X^p du\right)^{1/p}.$$

*The smallest such $C$ is called the $\mathcal{R}$-bound of $\mathcal{T}$, which is denotes by $\mathcal{R}_{\mathcal{L}(X,Y)}(\mathcal{T})$.*

As the second step, we obtain the maximal regularity for the linearized system by means of the $\mathcal{R}$-boundedness obtained above and the operator-valued Fourier multiplier theorem due to Weis [1].

Finally, we prove the local solvability of the Navier-Stokes-Korteweg system in the maximal regularity framework.

## References

[1] L. Weis, Operator-valued Fourier multiplier theorems and maximal $L_p$-regularity, Math. Ann., **319** (2001), 735–758.

— — —

Kiyoshi Saito
Department of Applied Mechanics and Aerospace Engineering, Waseda University, Tokyo, Japan

**Study on Two-Phase Flow Phenomena in Vapor Absorption/Compression Heat Pump**

In recent years, a heat-pump technology attracts attention as an innovative technology to realize energy saving in various fields, both of civilian and industrial sectors. The heat pump is a system which follows a reverse Carnot cycle as the basic principle. Since the system efficiency is high and also the configuration is comparatively simple, the vapor compression heat pump technology is spreading all over the world.

Various two-phase flow phenomena occur inside the heat pump; evaporation, condensation, expansion, phase separation, wetting, high speed flow, absorption, and so on. From the engineering viewpoint, various researches on these problems have been carried out over many years.

In this report, we would like to introduce some parts of these efforts and to share these problems with you all. First of all, an absorption heat and mass transfer phenomenon is discussed by showing some experimental results. Understanding of the absorption phenomenon occurring inside an absorption heat pump is significantly important to enhance the system efficiency. Secondly, we discuss the phase separation. Separating technique of gas-liquid two-phase flow into each phase is very important to enhance the system efficiency. However, as you know, complete separation of two-phase flow is extremely difficult. Some visualization results with high-speed camera are also shown.

— — —

YOSHIHIRO SHIBATA AND KEIICHI WATANABE
DEPARTMENT OF MATHEMATICS, WASEDA UNIVERSITY, TOKYO, JAPAN

**On the Navier-Stokes-Korteweg equations**

I will talk about some modelling about the Navier-Stokes-Korteweg equations from the point of Korteweg tensors. And, I will explain a result due to Keiichi Watababe concerning the incompressible and compressible two phase problem separated by a sharp interface with phase transition.

— — —

YUKIHITO SUZUKI
ORGANIZATION FOR UNIVERSITY RESEARCH INITIATIVES, WASEDA UNIVERSITY, TOKYO, JAPAN

**A GENERIC formalism and the interstitial work flux for Korteweg-type fluids**

In this talk, a GENERIC[1, 2] formalism for Korteweg-type fluids will be presented. The interstitial working introduced by Dunn and Serrin[3] is appears naturally in the formulation and proves to be necessary if we claim that the work done by the Korteweg stress is “isentropic”. The relation of the interstitial working to the micro-forces introduced by Gurtin[4] is also discussed.

## References


[1] Grmela, M., Öttinger, H. C., Dynamics and thermodynamics of complex fluids. I. Development of a general formalism, Phys. Rev. E 56, 6620-6632 (1997).

[2] Öttinger, H. C., Grmela, M., Dynamics and thermodynamics of complex fluids. II. Illustrations of a general formalism, Phys. Rev. E 56, 6633-6655 (1997).

[3] Dunn, J. E., Serrin, J., On the thermomechanics of interstitial working, Arch. Rational Mech. Anal. 88, 95-133 (1985).

[4] Gurtin, M. E., Generalized Ginzburg-Landau and Cahn-Hilliard equations based on a microforce balance, Physica D 92, 178-192 (1996).


— — —


Kenji Takizawa

Department of Modern Mechanical Engineering, Waseda University, Tokyo, Japan


### Space-Time Computational Analysis with Isogeometric Discretization

This presentation is an overview of how NURBS basis functions in space and time are enabling Space–Time Computational Analysis (STCA) to bring solution to a diverse set of challenging engineering problems. With the integration of the core ST [1] and NURBS technologies, we not only increase what we can do with the ST methods [2] and how accurately we can do it, but we also get more out of using NURBS basis functions in space [3, 4]. Integration with additional ST technologies such as the ST Slip Interface [2] and ST Topology Change [3] methods further increase the scope and accuracy of our overall STCA technology [7, 5]. The examples presented will include engineering and biomedical problems.

## References


[1] K. Takizawa and T.E. Tezduyar, Multiscale space-time fluid-structure interaction techniques, *Computational Mechanics*, **48**, 247-267, 2011.

[2] K. Takizawa and T.E. Tezduyar, Space–Time Fluid–Structure Interaction Methods, *Mathematical Models and Methods in Applied Sciences*, **22**, 1230001, 2012.

[3] K. Takizawa, T.E. Tezduyar, Y. Otoguro, T. Terahara, T. Kuraishi and H. Hattori, Turbocharger flow computations with the Space-Time Isogeometric Analysis (ST-IGA), *Computers & Fluids*, **142**, 15-20, 2017.

[4] Y. Otoguro, K. Takizawa and T. E. Tezduyar, Space time VMS computational flow analysis with isogeometric discretization and a general-purpose NURBS mesh generation method, *Comupers & Fluids*, 10.1016/j.compfluid.2017.04.017.

[5] K. Takizawa, T.E. Tezduyar, H. Mochizuki, H. Hattori, S. Mei, L. Pan and K. Montel, Space-time VMS method for flow computations with slip interfaces (ST-SI), *Mathematical Models and Methods in Applied Sciences*, **25**, 2377-2406, 2015.

[6] K. Takizawa, T.E. Tezduyar, A. Buscher and S. Asada, Space-time interface-tracking with topology change (ST-TC), *Computational Mechanics*, **54**, 955-971, 2014.

[7] K. Takizawa, T.E. Tezduyar, S. Asada and T. Kuraishi, Space-time method for flow computations with slip interfaces and topology changes (ST-SI-TC), *Computers & Fluids*, **141**, 124-134, 2016.

[8] K. Takizawa, T. E. Tezduyar, T. Terahara and T. Sasaki, Heart valve flow computation with the integrated Space–Time VMS, Slip Interface, Topology Change and Isogeometric Discretization methods, *Comupers & Fluids*, 10.1016/j.compfluid.2016.11.012.

— — —

Kazuyuki Tsuda
Department of Mathematics, Osaka University, Osaka, Japan

### Time decay estimate with diffusive property and smoothing effect for solution to the compressible Navier-Stokes-Korteweg system

Time decay estimate of a solution to the compressible Navier-Stokes-Korteweg system is considered. Concerning the linearized problem, the decay estimates with diffusive property for initial date are derived. As an application, the time decay estimates of a solution to the nonlinear problem are given. In contrast to the compressible Navier-Stokes system, for linear system regularities of initial dates are lower and independents of the order of derivative of the solution owing to smoothing effect from the Korteweg tensor. Furthermore, for the nonlinear system diffusive properties are obtained with initial dates having lower regularity than that of studies of the compressible Navier-Stokes system. This talk is based on a joint work with Prof. Takayuki Kobayashi (Osaka University).

— — —

Akifumi Yamaji
Cooperative Major in Nuclear Energy, Graduate School of Advanced Science and Engineering, Waseda University, Tokyo, Japan

### Application of Moving Particle Semi-implicit Method (MPS) for Analysis of Molten Core Behavior in Severe Accident of Nuclear Reactors

Moving Particle Semi-implicit Method (MPS) is a Lagrangian based particle method for incompressible flows. The basic governing equations are Navier-Stokes equation with mass and energy conservation equations and these equations are discretized by the following particle interaction models with a weight function which determines interaction distance of the particles (calculation points):

$$\text{Gradient model}: \langle \nabla \phi \rangle_i = \frac{d}{n^0} \sum_{j \neq i} \frac{\phi_j - \phi_i}{|\mathbf{r}_j - \mathbf{r}_i|^2} (\mathbf{r}_j - \mathbf{r}_i) w(|\mathbf{r}_j - \mathbf{r}_i|)$$

$$\text{Divergence model}: \langle \nabla \cdot \varphi \rangle_i = \frac{d}{n^0} \sum_{j \neq i} \left[ \frac{(\varphi_j - \varphi_i)(\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^2} w(|\mathbf{r}_j - \mathbf{r}_i|) \right]$$

$$\text{Laplacian model}: \left\langle \nabla^2 \phi \right\rangle_i = \frac{2d}{\lambda n^0} \sum_{j \neq i} (\phi_j - \phi_i) w(|\mathbf{r}_j - \mathbf{r}_i|)$$

This method is suitable for tracking evolution of free surfaces involving solid-liquid phase changes such as melting and solidification. The phase change is currently modeled by " solid fraction " as a function of enthalpy. Some additional models incorporated in the current MPS method for analysis of melting/solidification includes models for: surface tension, heat conduction, convection (buoyancy), radiation, turbulence (Large Eddy Simulation), viscosity change, etc. The developed method has been extensively tested and applied for predicting molten core behavior during severe accident of a nuclear reactor. For example, the MPS method has been used for predicting " spreading " of the discharged molten core in the damaged reactor building. The method has also been applied for predicting a long term Molten Core Concrete Interaction (MCCI). Both phenomena can be characterized as a tracking free surface problem, where the Lagrangian based MPS method has great advantage over the classical Eulerian based method.

**KEYWORDS**

*Moving Particle Semi-implicit Method (MPS), particle method, Lagrangian method, incompressible flow, phase change.*

— — —

Thomas Eiter
Department of Mathematics, Technical University Darmstadt, Darmstadt, Germany

### Falling drop in an unbounded liquid reservoir: Steady-state solutions

The equations of motion of a liquid drop in an unbounded liquid reservoir are discussed. The fluid behavior inside and outside the drop is modeled by the Navier–Stokes equations, which are coupled by certain boundary conditions. The unknown free boundary of the drop is described by a height function $\eta$ on the unit sphere.

One of the boundary conditions includes the mean curvature of the free boundary, which leads to a differential equation for $\eta$, whose linearization is given by a resolvent problem for the Laplace–Beltrami operator on the unit sphere. Moreover, the height function $\eta$ provides a corresponding coordinate transformation, which allows to formulate the equations in a fixed domain and to investigate the problem in a setting of Sobolev-type spaces.

The only prescribed (smallness) parameter is the difference $\rho_1 - \rho_2$ of the densities of the fluids. In particular, the Reynolds number is a free parameter of the problem. To obtain a well-posed linearization, the system is linearized around an appropriate steady state, which leads to an Oseen-type system. An application of the contraction mapping principle then yields a solution for the nonlinear problem when $|\rho_1 - \rho_2|$ is sufficiently small.

— — —

Takanori Kuroda
Department of Pure and Applied Physics, Waseda University, Tokyo, Japan

### The Asymptotic Analysis on a Complex Ginzburg-Landau Equation Based on the Potential-Well Method

In this talk, we investigate the asymptotic behavior of the solutions for the following complex Ginzburg-Landau equation:

$$\begin{cases} u_t(t,x) - e^{i\theta}[\Delta u + |u|^{q-2}u] - \gamma u = 0 & (t,x) \in [0,+\infty) \times \Omega, \\ u(t,x) = 0 & (t,x) \in [0,+\infty) \times \partial\Omega, \\ u(0,x) = u_0(x) & x \in \Omega, \end{cases} \tag{CGL}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary; $i = \sqrt{-1}$ is the imaginary unit; $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $\gamma \in \mathbb{R}$ and $q > 2$ are parameters. The above equation is a special form of the following with general coefficients $\lambda > 0$, $\kappa, \alpha, \beta, \gamma \in \mathbb{R}$:

$$u_t(t,x) - (\lambda + i\alpha)\Delta u - (\kappa + i\beta)|u|^{q-2}u - \gamma u = 0. \tag{CGL$'$}$$

The equation (CGL)$'$ was originally introduced as the mathematical model of superconductivities by Landau and Ginzburg in 1950. It is known that (CGL)$'$ describes also chaotic phenomena in fluid dynamics.

Levermore and Oliver ([3]) investigated the well-posedness and regularity for the solution of (CGL)$'$ with $\kappa < 0$ and with periodic boundary conditions in analogy with the Navier-Stokes equation. On the other hand, we have shown the local in time well-posedness for the case $\kappa > 0$ using the parabolic equation theory with non-monotone perturbation developed by Ôtani [4].

As for the asymptotic analysis, it is known that there exists a Lyapunov functional for (CGL) when $\gamma = 0$ and the finite time blow-up for initial data with negative energy is presented by Cazenave-Dickstein-Weissler [2] when $\gamma = 0$ and by Cazenave-Dias-FIgueira [1] when $\gamma \neq 0$.

We derive a Lyapunov functional for (CGL) when $\gamma \neq 0$ by changing the phase of the solutions. Using the Lyapunov functional, we apply the so-called potential-well method to prove the finite time blow-up with non negative initial date under the suitable condition on $\gamma$.

This talk is based on the joint work with Professor Ôtani at Waseda University.

## References


[1] Cazenave, T.; Dias, J.; Figueira, M., *J. Evol. Equ.* **14** (2014), no. 2, 403-415.

[2] Cazenave, T.; Dickstein, F.; Weissler, B., *SIAM J. Math. Anal.* **45** (2013), no. 1, 244-266.

[3] Levermore, D.; Oliver, M., *Dynamical systems and probabilistic methods in partial differential equations*, Berkeley, CA, (1994), 141-190, *Lectures in Appl. Math.* **31**, Amer. Math. Soc., Providence, RI (1996).

[4] Ôtani, M., *J. Differential Equations* **46** (1982), no. 2, 268-299.


— — —


Miho Murata

Department of Mathematics, Kanagawa University, Yokohama, Japan


### The global well-posedness for the compressible fluid model of Korteweg type


n this talk, we consider the Navier-Stokes-Korteweg system which was first introduced by D.J.Korteweg in 1901. It is shown that the system admits a unique, global strong solution for small initial data in $\mathbb{R}^N$, $N \geq 2$. For the purpose, the main tools are the maximal $L_p$-$L_q$ regularities and $L_p$-$L_q$ decay properties to the linearized equations. This talk is based on a joint work with Professor Yoshihiro Shibata in Waseda University.


— — —


Takuya Terahara

Department of Modern Mechanical Engineering, Waseda University, Tokyo, Japan


### Heart Valve Flow Analysis with the Integrated Space–Time Variational Multiscale, Slip Interface, and Topology Change Methods and Isogeometric Discretization

Heart valve flow analysis requires accurate representation of boundary layers near moving surfaces, even when the leaflets come into contact, and handling high geometric complexity. We address these challenges with a space–time (ST) method that integrates three ST methods in the framework of the ST Variational Multiscale (ST-VMS) [1] method: the ST Slip Interface (ST-SI) [2], ST Topology Change (ST-TC) [3] methods and ST Isogeometric Analysis (ST-IGA) [4]. The computations are for a realistic aortic-valve model with prescribed leaflet motion. The ST-VMS, as a moving-mesh method, maintains high-resolution boundary layer representation near solid surfaces. The ST-TC enables moving-mesh computation with TC, such as contact between the leaflets, maintaining high-resolution representation near the leaflets. The ST-SI was introduced for high-resolution representation near spinning surfaces. The mesh covering a spinning surface spins with it, and the SI between the spinning mesh and the rest accurately connects the two sides. For heart valves, the SI connects the mesh sectors containing the leaflets, enabling a more effective mesh moving. Integrating the ST-IGA with the ST-SI and ST-TC increases flow solution accuracy while keeping the element density in narrow spaces near contact areas at a reasonable level. The computations show the effectiveness of the ST-SI-TC-IGA method [5].

# References


[1] K. Takizawa and T.E. Tezduyar, Multiscale space-time fluid-structure interaction techniques, *Computational Mechanics*, **48**, 247-267, 2011.

[2] K. Takizawa, T.E. Tezduyar, H. Mochizuki, H. Hattori, S. Mei, L. Pan and K. Montel, Space-time VMS method for flow computations with slip interfaces (ST-SI), *Mathematical Models and Methods in Applied Sciences*, **25**, 2377-2406, 2015.

[3] K. Takizawa, T.E. Tezduyar, A. Buscher and S. Asada, Space-time interface-tracking with topology change (ST-TC), *Computational Mechanics*, **54**, 955-971, 2014.

[4] K. Takizawa, T.E. Tezduyar and T. Terahara, Ram-Air Parachute Structural and Fluid Mechanics Computations with the Space–Time Isogeometric Analysis (ST-IGA), *Computers & Fluids*, **141**, 191-200, 2016.

[5] K. Takizawa, T. E. Tezduyar, T. Terahara and T. Sasaki, Heart valve flow computation with the integrated Space–Time VMS, Slip Interface, Topology Change and Isogeometric Discretization methods, *Comupers & Fluids*, 10.1016/j.compfluid.2016.11.012.


— — —


Shun Uchida

Department of Applied Physics, Waseda University, Tokyo, Japan


## Existence of periodic solutions to some system describing double-diffusive convection phenomena

We consider the time periodic problem of the following system which describe double-diffusive convection phenomena in some porous medium.

$$\begin{cases} \partial_t \vec{u} = \nu \Delta \vec{u} - a\vec{u} - \nabla p + \vec{g}T + \vec{h}C + \vec{f_1} \\ \partial_t T + \vec{u}\cdot\nabla T = \Delta T + f_2 \\ \partial_t C + \vec{u}\cdot\nabla C = \Delta C + \rho \Delta T + f_3 \\ \nabla\cdot\vec{u} = 0 \end{cases} \quad \text{in } \mathbb{R}^N \times [0, S],$$

where $N = 3, 4$ and $S > 0$ denotes the time period. Unknown functions are $\vec{u}$, $T$, $C$ and $p$, which represent the fluid velocity, the temperature, the concentration of solute and the pressure, respectively. As for given data, $a$, $\nu$, $\rho$ are positive constants and $\vec{g}$, $\vec{h}$ are constant vectors. Moreover, $\vec{f_1}$, $f_2$, $f_3$ designate given external forces. The main purpose of this talk is to construct a periodic solution of the system without the smallness condition of given data $\vec{f_1}$, $f_2$, $f_3$ via the convergence of solutions for some approximate equations in bounded domains.

— — —

Xin Zhang
Department of Mathematics, Waseda University, Tokyo, Japan

## On the persistence of Hölder regular patches of density for the inhomogeneous Navier-Stokes equations

In this short talk, we will present some recent result concerning long time persistence of $C^{1,\varepsilon}$ regular density patch for inhomogeneous Navier-Stokes equation in the framework of critical spaces, which is a joint work with Raphaël Danchin. Our proof is based on the fundamental idea from Jean-Yves Chemin in the work of vortex patch problem for 2-D incompressible Euler equations in 1990s. Besides, we also want to review other recent contributions with employing supercritical velocity fields.

— — —