International Workshop on the Multi-Phase Flow; Analysis, Modeling and Numerics

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Abstracts

TOP GLOBAL UNIVERSITY JAPAN

科研費 KAKENHI

早稲田大学 WASEDA University

Thomas Richter
Faculty of Mathematics, University of Magdeburg, Magdeburg, Germany

Finite Elements for Fluid-Structure Interactions

In this lecture, we will present numerical tools for the simulation of coupled fluid-structure interaction problems. Fluid-structure interactions are part of many application problems in engineering, biology and medicine. We focus on the strongly coupled interaction of an incompressible fluid with a nonlinear elastic structure. Usually the coupling is be considered to be strong, if fluid and solid mass are comparable (like the coupling of blood with tissue and unlike the coupling of air and steel). Modeling of such problems leads to stiffly coupled nonlinear systems of partial differential equations. Many standard simulation tools will fail.

Part 1. Monolithic schemes in Arbitrary Lagrangian Eulerian Coordinates
We start by describing the ALE scheme for fluid-structure interactions, which is based on the mapping of the moving fluid domain onto a fixed reference domain. This framework allows to present the fully coupled system in one closed variational formulation, making it accessible for many well-established tools like Newton's method, multigrid, a posteriori error estimation and gradient based optimization are available.

Part 2. Large deformation problems in the Fully Eulerian Formulation
ALE formulations for fluid-structure interactions experience a limit in terms of large movement and deformation. They usually do not allow for very large deformation and they forbid topology change, e.g. by contact or by material failure. We present an alternative approach, the Fully Eulerian framework, that is based on a mapping of all equations into the moving Eulerian coordinate framework. The resulting system looks similar to multi-phase flows and brings along comparable difficulties. The major numerical problem is the moving-interface character with an interface line (or surface) between fluid and structure, which must be captured and which must be resolved be the discretization. The main focus of this lecture is to present spatial and temporal discretization schemes for interface problems, that are also useful for comparable interface problems with moving interfaces.

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Giovanni Paolo Galdi
Department of Mechanical Engineering and Materials Science,
Department of Mathematics, University of Pittsburgh, Pittsburgh, USA

Flow of a Viscous Liquid Past an Obstacle at Arbitrary Reynolds Number

In this short two-hour course, I shall perform a quick ride through one of the oldest and most investigated problems in fluid mechanics, namely, the motion of a viscous liquid past a fixed rigid obstacle. Assuming the flow is uniform at large spatial distance (infinity) from the body, I will mainly focus on the determination of the motion characteristics at Reynolds numbers, Re, of arbitrary size. Thus, besides basic issue like existence and regularity, this study will include the occurrence of (multiple) steady and time-periodic bifurcating phenomena, along with a quick (and, unfortunately, to date still incomplete) glance at the behavior in the limit of very large Re. Certain significant open questions will also be identified and discussed.

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Franco Flandoli

Department of Mathematics, University of Pisa, Pisa, Italy

Mean field results for particles dispersed in fluids

The talk will present two attempts to model a large family of small particles dispersed in a fluid. Both models aim to be simple and leave open difficult questions about a very accurate modelling of the two phase flow.

The first model considers the case of interacting particles embedded into a "turbulent" fluid, in the very simplified regime when the particles are passive and the fluid is represented by a space-time Gaussian field, white in time, in the spirit of the theory of passive scalars in turbulent fluids. We show a mean field result, in the limit when the number of particles goes to infinity and their interaction is of mean field type, which give rise to a stochastic nonlinear PDE.

The second model removes both idealizations, namely the Gaussian field is replaced by the solution of the Navier-Stokes equations and the particles act on the fluid. However, we idealize the interaction between particles and fluid, having in mind particles of infinitesimal size and Stokes drag force. The limit PDE is of Vlasov-Navier-Stokes type. We compare methodologies and models and address open questions.

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Yoshihiro Shibata

Department of Mathematics, Waseda University, Tokyo, Japan

On some free boudary problem for the Navier-Stokes equations

I will talk about some free boundary problem for the Navier-Stokes equations. Contents is the following.

1. Modelling of the free boundary problem for the Navier-Stokes equations.
2. Maximal L_p-L_q regularity and Local well-posedness.
3. Global well-posedness in the bounded domain with surface tension.
4. Global well-posedness in the exterior domain case without surface tension.

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Suncica Canic
Department of Mathematics, University of Houston, Texas, USA

An Introduction to Fluid-Solid Interaction in Biofluids

Fluid-solid interaction (FSI) problems arise in many applications. They include multi-physics problems in engineering such as aeroelasticity and propeller turbines, as well as biofluidic application such as self-propulsion organisms, fluid-cell interactions, and the interaction between blood flow and cardiovascular tissue. A comprehensive study of these problems remains to be a challenge due to their strong nonlinearity and multi-physics nature. To make things worse, in many biological applications the solid is composed of several layers, each with different mechanical characteristics. This is, for example, the case with arterial walls whose physiology and pathophysiology are affected by their interaction with blood flow, and by the interaction between the different arterial wall layers. In the two introductory lectures, the speaker will present an overview of the main problems in FSI in blood flow motivated by cardiovascular applications. In particular, mathematical modeling, analysis, and design of partitioned numerical algorithsm for FSI in hemodynamics will be discussed.

Lecture 1: Modeling of Fluid-Solid Interaction in Hemodynamics

In this introductory lecture, models for blood flow and arterial walls will be presented. The coupling between an incompressible, viscous fluid modeling blood flow, and an elastic/viscoelastic solid modeling arterial walls will be described. The geometric nonlinearities in the coupled problem will be discussed, and the reasons for the instabilities in the classical partitioned (loosely-coupled) FSI schemes will be explained. Issues related to the use of " correct " boundary conditions will be touched. Modeling of vascular devices such as stents using the 1D hyperbolic net approaches will be presented.

Lecture 2: Analysis and a Partitioned Numerical Scheme for Fluid-Solid Interaction in Hemodynamics

The speaker will present an overview of the mathematical analysis results for fluid-solid interaction with application to blood flow, and an example of a partitioned numerical scheme, which inspired a constructive existence proof for this class of problems, will be discussed.

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Elena Frolova

St. Petersburg Electrotechnical University, St. Petersburg State University, St. Petersburg, Russia

Free boundary problem of magnetohydrodynamics for two liquids

We consider the free boundary problem governing the motion of two viscous incompressible electrically conducting capillary fluids separated by a closed interface. Media is moving under the action of magnetic field. We assume that the first fluid is contained in a bounded variable domain Ω_{1t}, which is surrounded by a bounded variable domain Ω_{2t}, filled with the second fluid. The boundary of Ω_{2t} consists of two disjoint components: the free boundary $\Gamma_t = \partial\Omega_{1t}$ and the fixed surface S. Both Γ_0 and S are homeomorphic to a sphere, $dist\{\Gamma_0, S\} \geq \delta > 0$. The surface S is assumed to be a perfect conductor.

We prove local in time solvability of this problem in Sobolev-Slobodetskii spaces $W_2^{2+l,1+l/2}$, $1/2 < l < 1$. To reduce the free boundary problem for a problem in a fixed domain, we use Hanzawa coordinate transform. The proof is carried out by the successive approximations method and based on the careful analysis of the corresponding linear problem and estimates of nonlinear terms.

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Tadahisa Funaki

Graduate School of Mathematical Sciences, University of Tokyo, Tokyo, Japan

Sharp interface limit for a stochastically perturbed mass conserving Allen-Cahn equation

We will discuss the sharp interface limit for a mass conserving Allen-Cahn equation added an external mild stochastic noise. In the limit, a stochastically perturbed volume preserving mean curvature flow is derived. Our approach is the asymptotic expansion method, in which powers of derivatives of the noise appear and these terms are diverging. However, making the convergence speed of our mild noise to the white noise slow enough, we can control them. This is joint work with Satoshi Yokoyama. The paper is posted on the arXiv.

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Matthias Hieber

Department of Mathematics, Technical University Darmstadt, Darmstadt, Germany

Strong Solutions to Two-Phase Free Boundary Value Problems for Newtonian and Non-Newtonian Fluids

In this talk we discuss one- and two-phase free boundary value problems subject to surface tension and gravitational forces for various classes of Newtonian and non- Newtonian fluids and discuss their well-posedness properties within the L^p-setting. The fluids considered are special classes of so called Stokesian fluids. We show in particular that for given $T > 0$ this type of problems admit a unique, strong solution on $(0, T)$ provided the initial data are sufficiently small in their natural norms. This is partly joint work with Hirokazu Saito (Waseda University).

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Toshiaki Hishida

Graduate School of Mathematics, Nagoya University, Nagoya, Japan

Navier-Stokes flow past a rigid body: attainability of steady solutions as the limit of unsteady weak solutions, starting and landing cases

Consider the Navier-Stokes flow in 3D exterior domains, where a rigid body is translating with prescribed translational velocity $-h(t)u_\infty$ with constant vector $u_\infty \in \mathbb{R}^3 \setminus \{0\}$. Finn raised the question whether his steady solutions are attainable as the limit for $t \to \infty$ of unsteady solutions starting from motionless state when $h(t) = 1$ after some finite time and $h(0) = 0$ (starting problem). This was affirmatively solved by Galdi, Heywood and Shibata for small u_∞. In this talk, we study some generalized situation in which unsteady solutions start from large motions being in L^3. We then conclude that the steady solutions for small u_∞ are still attainable as the limit of evolution of those fluid motions which are found as a sort of weak solutions. The opposite situation, in which $h(t) = 0$ after some finite time and $h(0) = 1$ (landing problem), is also discussed. In this latter case, the trivial solution is attainable in the same way as above no matter how large u_∞ is. This talk is based on a joint work with Paolo Maremonti (Seconda Universita degli Studi di Napoli).

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Yoshifumi Kimura

Graduate School of Mathematics, Nagoya University, Nagoya, Japan

Search for a standard model of vortex reconnection

As a fundamental process in the whole vortex problems, vortex reconnection has been studied intensively and providing renewed interest in both theoretical and computational fluid mechanics. In this talk, inspired by recent novel experiment by Kleckner and Irvine on the dynamics of a trefoil knot vortex [1], we first present a linearized model in which two Burgers-type vortices are driven together by an axisymmetric straining velocity field [2]. Extending this linearized model, we then consider the nonlinear vortex interaction using the Biot-Savart integral. It will be demonstrated that there are clear scaling properties in time for the minimum distance of anti-parallel vortices, the maximum velocity and the maximum axial strain rate before reconnection, and that specific shapes of vortices are formed near during the reconnection.

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Yasunori Maekawa
Department of Mathematics, Kyoto University, Kyoto, Japan

On some stability results for two-dimensional exterior flows

In this talk recent progress will be introduced on the stability theory for some typical viscous incompressible flows in a two-dimensional exterior domain. Particular interests are: time-dependent weak L^2 flows including forward self-similar type such as the Lamb-Oseen flows, stationary circular flows decaying in a scale-critical order, and Oseen flows.

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Anne Robertson
Department of Mechanical Engineering and Materials Science,
Department of Bioengineering, University of Pittsburgh, Pittsburgh, USA

Modeling the mechanobiology of the arterial wall: Our current understanding and future research directions

The mechanical strength of the artery wall is vital to the health of the individual. This integrity is in turn dependent on the state of the central load bearing components of the wall: collagen fibers, elastin fibers and vascular smooth muscle cells. In healthy arteries, the physical organization and mechanical properties the elastin and collagen fibers enable the artery wall to withstand the mechanical loads created by the time periodic pulsation of the blood, without excessive energy dissipation and without exposing the cells in the wall to inordinate strains. Furthermore, these fibers must be able to withstand on the order of 10^9 loading cycles over the course of a seventy-five year old lifetime without excessive fatigue. Fortunately, collagen fibers, which are responsible for the wall strength, are not simply created at birth and then left to slowly wear out over time. Rather, the cells in the artery wall are designed to continually renew these collagen fibers. In fact, is it now understood that the healthy arterial wall is an active structure- with collagen fibers undergoing growth and remodeling in response to blood flow (hemodynamic) derived chemical and mechanical stimuli. This replacement process also adds an element of flexibility to the wall, enabling it to change the collagen fiber organization in response to altered flow and mechanical loads.

In this talk, we discuss mathematical models that have been developed to describe this coupled response between the mechanics and the biology (mechanobiology) as well as the somewhat universal factors that appear to dominate the inherent optimization of collagen fiber organization. Of great interest are theoretical frameworks that are capable of modeling the response of the artery wall to mechanical and chemical perturbations to the flow and mechanical loading. In this talk, we discuss some of these mathematical frameworks with particular focus on the i) prototypical healthy artery wall, ii) aneurysms of brain arteries, and iii) in vivo tissue engineering of neoarteries. Recent advances in bioimaging enable unprecedented data on the collagen fiber architecture. For example, multi-photon microscopy (MPM) can be used to visualize both elastin and collagen in artery walls by exploiting auto-fluorescence and second harmonic generation, respectively. In this talk, we discuss bioimaging results obtained in our laboratory using MPM that are used to drive these theoretical models. The talk will also include a discussion of open questions, future directions and the need for further collaborations with mathematicians, biologist and engineers.

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Kiyoshi Saito
Department of Applied Mechanics and Aerospace Engineering, Waseda University, Tokyo, Japan

Experimental Study on Two-Phase Flow in Vapor Compression Heat Pump

In recent years, heat-pump technology attracts attention as an innovative technology to realize energy saving in various fields, both of civilian and industrial sectors. A vapor compression heat pump is a system which follows a reverse Carnot cycle as the basic principle. Since the system efficiency is high and also the configuration is comparatively simple, the vapor compression heat pump technology is spreading all over the world. Various two-phase flow phenomena occur inside the heat pump; evaporation, condensation, expansion, phase separation, wetting, high speed flow and so on. From the engineering viewpoint, various researches on these problems have been carried out over many years. In this report, we would like to introduce some parts of these efforts and to share these problems with you all. First of all, flow boiling heat transfer inside small channels is discussed by showing some experimental results. A small channel is defined as a channel with a diameter smaller than 1 mm, and heat transfer inside these channels has recently become a hot topic in the engineering field. This is because their heat transfer performance is significantly higher than that of conventional channels, and this characteristics can reduce the required charge of working fluid. A small channel shows heat transfer characteristics different from a conventional channel. Secondly, we discuss experimental results of phase separation in a T-shaped junction. Separating technique of gas-liquid two-phase flow into each phase is very important to enhance the system efficiency. However, as you know, complete separation of two-phase flow is extremely difficult. Some visualization results with high-speed camera are also shown. We hope that this discussion will contribute establishing collaborations beyond this specific research field and to realizing an energy saving society.

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Franck Sueur

Institute of Matematics, University of Bordeaux, Bordeaux, France

Point vortex dynamics as zero-radius limit of the motion of rigid bodies in a perfect incompressible fluid

The vortex point system is usually considered as an idealized model where the vorticity of an ideal incompressible two-dimensional fluid is concentrated in a finite number of moving points. In this talk I will present some results obtained in collaboration with O. Glass, C. Lacave and A. Munnier which establish that point vortex dynamics can also be obtained as zero-radius limit of rigid bodies motions under the influence of the force exerted by the fluid pressure on its boundary.

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Yoshie Sugiyama

Department of Mathematics, Kyushu University, Fukuoka, Japan

Time global existence and finite time blow-up criterion for solutions to the Keller-Segel system coupled with Navier-Stokes fluid

We will deal with the chemotaxis model under the effect of the Navier-Stokes fluid, *i.e.*, the incompressible viscous fluid. We shall show the existence of a local *mild solution* for large initial data and a global *mild solution* for small initial data in the scale invariant class demonstrating that $n_0 \in L^1(\mathbb{R}^2)$ and $u_0 \in L^2_\sigma(\mathbb{R}^2)$. Our method is based on the perturbation of linearization together with the $L^p - L^q$-estimates of the heat semigroup. As a by-product of our method, we shall prove the smoothing effect and uniqueness of our *mild solution*. In addition, we shall show a blow-up criterion which almost covers the well-known threshold number 8π of the size $||n_0||_{L^1(\mathbb{R}^2)}$ under the rest state of the fluid motion. Furthermore, the blow-up rate will be also discussed. This is based on a joint work with Professor Hideo KOZONO(Waseda university) and Mr. Masanari MIURA (Kyushu university).

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Hiroshi Suito

Graduate School of Environmental and Life Science, Okayama University, Okayama, Japan

Fluid dynamical and statistical analyses for morphological characteristics of large arteries

We present numerical experiments related to cardiovascular problems such as the aortic aneurysm, which is one of the life-threatening diseases. In this study, morphological characterization of blood vessels, which has wide variety among individuals, is introduced. Differences in the morphology of the vessels bring about differences in the flow characteristics, which lead to different outcomes.

We consider a number of patient-specific models of the aorta as constructed from CT scans. We compute the flow fields with Space–Time Variational Multiscale (ST-VMS) method[1–4].

This work was supported by JST-CREST mathematics program.

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Kenji Takizawa

Department of Modern Mechanical Engineering, Waseda University, Tokyo, Japan

Space-Time Computational Analysis with NURBS in Space and Time

Space-Time (ST) Variational Multiscale (ST-VMS) method [1] and its predecessor ST-SUPS [2] have a good track record in computational analysis of complex fluid-structure interactions (FSI) and flows with moving boundaries and interfaces (MBI). When an FSI or MBI problem requires high-resolution representation of boundary layers near solid surfaces, ALE and ST methods, where the mesh moves to follow the fluid-solid interface, meet that requirement. Moving-mesh methods have been practical in more classes of complex FSI and MBI problems than commonly thought of. With a number of complementary methods introduced recently, the ST methods can now do even more than that. They can deal with contact between solid surfaces or other topology changes, as enabled by the ST-TC method [3], or a spinning solid surface that is in contact with a solid surface, as enabled by the ST Slip Interface TC (ST-SI-TC) method [4]. Using NURBS as basis functions is further increasing the accuracy and scope of the ST methods [5]. In the ST context, the options for using NURBS as basis functions are in space (ST-NS), in time (ST-NT), and in space and time (ST-NST). A general-purpose NURBS mesh generation method introduced recently makes the use of ST-NS and ST-NST options more practical in computations involving complex geometries. That practicality is further increased by the ST-SI method [6]. We present an overview of all these methods and some test computations.

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Akifumi Yamaji

Cooperative Major in Nuclear Energy, Graduate School of Advanced Science and Engineering, Waseda University, Tokyo, Japan

Application of Moving Particle Semi-implicit Method (MPS) for Analysis of Molten Core Behavior in Severe Accident of Nuclear Reactors

Moving Particle Semi-implicit Method (MPS) is a Lagrangian based particle method for incompressible flows. The basic governing equations are Navier-Stokes equation with mass and energy conservation equations and these equations are discretized by the following particle interaction models with a weight function which determines interaction distance of the particles (calculation points):

$$\text{Gradient model}: \langle \nabla \phi \rangle_i = \frac{d}{n^0} \sum_{j \neq i} \frac{\phi_j - \phi_i}{|\mathbf{r}_j - \mathbf{r}_i|^2} (\mathbf{r}_j - \mathbf{r}_i) w(|\mathbf{r}_j - \mathbf{r}_i|)$$

$$\text{Divergence model}: \langle \nabla \cdot \varphi \rangle_i = \frac{d}{n^0} \sum_{j \neq i} \left[\frac{(\varphi_j - \varphi_i)(\mathbf{r}_j - \mathbf{r}_i)}{|\mathbf{r}_j - \mathbf{r}_i|^2} w(|\mathbf{r}_j - \mathbf{r}_i|) \right]$$

$$\text{Laplacian model}: \left\langle \nabla^2 \phi \right\rangle_i = \frac{2d}{\lambda n^0} \sum_{j \neq i} (\phi_j - \phi_i) w(|\mathbf{r}_j - \mathbf{r}_i|)$$

This method is suitable for tracking evolution of free surfaces involving solid-liquid phase changes such as melting and solidification. The phase change is currently modeled by “ solid fraction ” as a function of enthalpy. Some additional models incorporated in the current MPS method for analysis of melting/solidification includes models for: surface tension, heat conduction, convection (buoyancy), radiation, turbulence (Large Eddy Simulation), viscosity change, etc. The developed method has been extensively tested and applied for predicting molten core behavior during severe accident of a nuclear reactor. There is particularly a great need to reveal the precise progression of the Fukushima accident and the current status of the damaged reactors for decommissioning planning (debris removal from the damaged reactors). Further development and improvement of the method is desired for mechanistic simulations, which do not rely on parameter adjustments (tuning). Some topics for further model improvements may include (miscellaneous): chemical reaction, eutectic formation, mixing, dissolution, gas generation.

KEYWORDS
Moving Particle Semi-implicit Method (MPS), particle method, Lagrangian method, incompressible flow, phase change, mechanistic analysis.

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Hiroaki Yoshimura
Department of Applied Mechanics and Aerospace Engineering, Waseda University, Tokyo, Japan

The Lagrangian formulation of nonholonomic mechanical systems

In this talk, we study the Lagrangian formulation of nonholonomic mechanical systems together with Lagrange-d ’ Alembert-Pontryagin principle. Specifically, we consider the case in which a configuration manifold is given by a Lie group together with the so-called Lie-Dirac reduction theory. Then, the Lagrange-Dirac systems for nonholonomic mechanical systems on Lie groups with broken symmetry will be studied to develop implicit Euler-Poincare-Suslov equations with advected parameters with some illustrative examples such as the Chaplygin ball and the second-order Rivlin-Ericksen Navier-Stokes fluids. We will also show some extensions of the Lagrangian variational formulation to the cases in which nonequilibrium thermodynamics is included by considering nonlinear nonholonomic constraints.

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Wojciech Zajaczkowski
Polish Academy od Science, Warsaw, Poland

On some free boundary problems to magnetohydrodynamics

We consider the magnetohydrodynamics motion in a domain bounded by a free boundary. In the exterior domain to this motion is generated electromagnetic field by some given magnetic field. On the free boundary we have the transmision conditions. First we show existence of local solutions. Next, under some smallness restrictions we prove global existence. We consider incompressible and compressible cases. We use the energy method, so L_2 approach. However we mention on some results in the L_p approach.

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Sebastian Elias Graiff Zurita
Balserio Institute, Rio Negro, Argentina

Discrete mechanical systems: existence of trajectories

In this talk we are going to see that discrete mechanical systems, viewed as discrete-time dynamical systems, behave in three different ways: given an initial condition such a system can have no solution, a unique solution, or multiple solutions. But, given a mechanical system, we can construct a one-parameter family of discrete mechanical systems that have a unique solution, in a neighborhood of a given initial condition, for all values of the parameter. Our construction uses the alternative formulation of discrete mechanical system given in [1], which is different from the usual construction (see, for example, [2]), but equivalent in some sense. Finally, we will discuss shortly the error analysis for these systems, and we will show that it is possible to extend all the analysis to discrete mechanical systems with forces.

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Kai Lee
Graduate School of Mathematical Sciences, University of Tokyo, Tokyo, Japan

Sharp interface limite for the stochastic Allen-Cahn equations

In this talk, we consider the stochastic Allen-Cahn equation with the Diriclet boundary condition;

$$\begin{cases} \dot{u}^\varepsilon(t,x) & = \Delta u^\varepsilon(t,x) + \dfrac{1}{\varepsilon} f(u^\varepsilon(t,x)) + \dot{W}_t^\varepsilon(x), \quad t>0,\ x\in[-1,1], \\ u^\varepsilon(0,x) & = u_0^\varepsilon(x),\ x\in\mathbb{R}, \quad u^\varepsilon(t,\pm 1)=\pm 1,\ t>0, \end{cases}$$

with a small parameter $\varepsilon > 0$. This equation describes a behavior of interface and the parameter ε corresponds to a width of interface. We are interested in the behavior of the solution u^ε when $\varepsilon \to 0$, and we call it the sharp interface limit. In this case, we can expect that the interface motion at the limit is described by a Brownian motion reflected at the boundary of $[-1,1]$. We prove it from the Mosco convergence of Diriclet form which is associated with the $L^2[-1,1]$-valued Markov process $u^\varepsilon(t)$. If we have more time, I would like to present about my previous work of "generation of interface".

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Hideki Murakawa

Faculty of Mathematics, Kyushu University, Fukuoka, Japan

Mathematics of cell-cell adhesion: modeling and analysis

Cell adhesion and cell sorting processes are essential in organ formation during embryonic development and in maintaining multicellular structure. Armstrong, Painter and Sherratt [J. Theor. Biol. 243 (2006), pp. 98–113] proposed a nonlocal advection-diffusion system as a possible continuous mathematical model for these phenomena. Although the system is attractive and challenging, it gives biologically unrealistic numerical solutions. We identify the problems and change underlying idea of cell movement from “cells move randomly” to “cells move from high to low pressure regions”. Then we provide a modified continuous model. Numerical experiments illustrate that the modified model is able to replicate not only the Steinberg’s cell sorting experiments but also some phenomena which can not be captured at all by Armstrong-Painter-Sherratt model. Furthermore, we give theoretical results about the modified model.

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Issei Oikawa

Department of Mathematics, Waseda University, Tokyo, Japan

A finite element method for the Stokes equations with a penalized slip boundary condition

In this talk, we present a penalty method for the Stokes equations with the slip boundary condition and its finite element approximation. When the domain is smooth, it is known that a variational crime occurs if the slip boundary condition is directly imposed. The penalty method enables us to avoid the variational crime and is easy to implement by popular finite element softwares, such as FreeFem and FEniCS. The error analysis of our method and numerical results will be presented. This talk is based on joint work with Takahito Kashiwabara and Guanyu Zhou.

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Kaori Onozaki

Department of Applied Mechanics and Aerospace Engineering, Waseda University, Tokyo, Japan

Trajectory design for space missions in the 4-body system

Low energy trajectories are required for space missions since resources are limited in the space. We will show the trajectory from the Earth to the Moon under the condition that

a spacecraft is influenced by gravities of the Sun, the Earth and the Moon. The 4-body system will be decoupled into two different 3-body systems with perturbations. Respective trajectory will be obtained in each perturbed system, and then the Earth-Moon trajectory will be constructed by connecting these trajectories based on the structures of the stable and unstable manifolds, which exist in both perturbed systems.

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Hirokazu Saito

Department of Mathematics, Waseda University, Tokyo, Japan

Free boundary problem of a compressible fluid model of Korteweg type

In this talk, we would like to consider a free boundary problem of a compressible fluid model of Korteweg type. Korteweg introduced in the early 1900s a new stress tensor $\mathbf{T} = \mathbf{S}+\mathbf{K}$ with $\mathbf{S} = \mu\mathbf{D}(\mathbf{v})+(\nu-\mu)\,\mathrm{div}\,\mathbf{v}\mathbf{I}$ and $\mathbf{K} = (\kappa/2)(\Delta\rho^2-|\nabla\rho|^2)\mathbf{I}-\kappa\nabla\rho\otimes\nabla\rho$, where $\mathbf{K}$ is called a Korteweg tensor nowadays, in order to treat phase transition phenomena. We in this talk deal with a model problem on $\mathbb{R}^N_+$, $N \geq 2$, arising from the free boundary problem. If there is time, then we will discuss the case of general domains.

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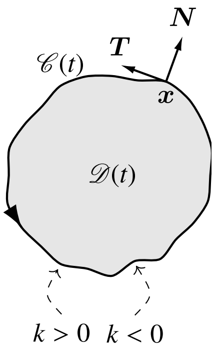
Koya Sakakibara

Graduate School of Mathematical Sciences, University of Tokyo, Tokyo, Japan

Structure-preserving numerical scheme for Hele-Shaw problems by the method of fundamental solutions

The classical Hele-Shaw problems is description of a motion of viscous fluid in a quasi two-dimensional space, which was starting from a short paper [1] in 1898 by Henry Selby Hele-Shaw (1854-1941). In his experiment, viscous fluid is sandwiched between two parallel plates with a narrow gap, and the apparatus is called Hele-Shaw cell. He succeeded to visualize streamlines by means of colored water in the cell. The classical Hele-Shaw problem is stated as follows:

$$
\begin{cases}
\triangle p(\cdot,t) = 0 & \text{in } \mathscr{D}(t),\ t \in [0,T), \\
p(\cdot,t) = \gamma k(\cdot,t) & \text{on } \mathscr{C}(t),\ t \in [0,T), \\
V(\cdot,t) = -\nabla p(\cdot,t) \cdot \boldsymbol{N}(\cdot,t) & \text{on } \mathscr{C}(t),\ t \in [0,T),
\end{cases}
\tag{0.1}
$$



where $\triangle$ and ∇ are the Laplace operator and the gradient in $\mathbb{R}^2$, respectively, $\mathscr{D}(t) \subset \mathbb{R}^2$ is a bounded region occupied by fluid, $\mathscr{C}(t)$ is the boundary of $\mathscr{D}(t)$ (positively oriented closed curve), $p(\cdot,t)$ is the pressure function in $\mathscr{D}(t)$, γ is the surface tension coefficient, $k(\cdot,t)$ is the curvature (sign convention is the way that $k = 1$ if $\mathscr{D}(t)$ is a unit disk), $\boldsymbol{N}(\cdot,t)$ is the unit outward normal vector defined by $\boldsymbol{N}(\cdot,t) = -\boldsymbol{T}^{\perp}(\cdot,t)$, and $V(\cdot,t)$ is the normal velocity, on $\mathscr{C}(t)$. See the right figure (in the figure, $\boldsymbol{x}$ is the position vector and $\boldsymbol{T}$ is the unit tangent vector). Here, $\boldsymbol{a} \cdot \boldsymbol{b}$ represents the usual two-dimensional Euclidean inner product for $\boldsymbol{a}, \boldsymbol{b} \in \mathbb{R}^2$, and $\boldsymbol{a}^{\perp} = (-b, a)^{\mathrm{T}}$ for $\boldsymbol{a} = (a, b)^{\mathrm{T}}$.

We have three marked properties of the classical Hele-Shaw problem (0.1): the total length of $\mathscr{C}(t)$ is decreasing in time, the enclosed area of $\mathscr{D}(t)$ is preserving and the barycenter of $\mathscr{D}(t)$ is being fixed. These properties are called curve-shortening (CS), area-preserving (AP) and barycenter-fixed (BF), respectively. Thus it is natural to consider that the numerical solution should satisfy these geometrical variational structures in some discrete sense, however, there does not exist such numerical scheme as far as we know. The purpose of this talk is that for (0.1) we propose a simple numerical scheme by means of the method of fundamental solutions combined with the uniform distribution method. As a result, we have succeeded to construct numerical scheme which satisfies CS-, AP- and BF-properties exactly, in which we have to solve system of "quadratic" equations, and the one which satisfies CS-, AP- and BF-properties asymptotically, in which we have to solve system of "linear" equations. Our scheme can also be applied for other vatiarions: the one-phase exterior problem, the one-phase interior Hele-Shaw problem with sink/source points, two-phase Hele-Shaw problem, and so on.

The contents of this talk are based on the joint work with Professor Shigetoshi Yazaki of Meiji University [2].

References

[1] H. S. Hele-Shaw, *The flow of water*, Nature **58** (1898), 34–36.

[2] K. Sakakibara and S. Yazaki, *Structure-preserving numerical scheme for the one-phase Hele-Shaw problems by the method of fundamental solutions*, submitted revised version.

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Yukihito Suzuki

Department of Mathematics, Waseda University, Tokyo, Japan

A bracket formulation and structure preserving numerical method for diffuse interface models

In this talk, a bracket formulation for the Navier-Stokes/Cahn-Hilliard equations, which employs a skew-symmetric Poisson bracket and a symmetric negative semidefinite dissipative bracket, will be proposed. We will also discuss a numerical method which properly inherits the conservation laws of energy, helicity and enstrophy derived from the bracket formulation, as well as the de Rham complex on a domain in thee dimensional Euclidean space. Some numerical examples will be shown.

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