Mathematics and Physics Unit “Multiscale Analysis, Modeling and Simulation”
Top Global University Project, Waseda University

International Workshop on “Fundamental Problems in Mathematical and Theoretical Physics”

Date: July 18 – July 22, 2016
Venue: Conference Room No.4, 2nd Floor, 55S Bldg., Waseda University, Nishi-Waseda Campus
早稲田大学 西早稲田キャンパス 55 号館 S 棟 2 階 第4会議室

Part II

◆Vladimir Georgiev (University of Pisa)◆

◆Mini Course Ⅰ	July 20, Wednesday	13:30 – 15:00
◆Mini Course Ⅱ	July 21, Thursday	10:30 – 12:00
◆Mini Course Ⅲ	July 22, Friday	10:30 – 12:00

Scattering of small solutions of the cubic NLS with short range potential

We consider 1-D Hamiltonian Schrödinger equation with cubic nonlinearity $|u|^2u$ and even short range potential. Under the (crucial) assumption for absence of resonances at zero, we show that small odd data gives rise to global solutions, which scatter at infinity at the rate of the free solution. This is a joint work with Atanas Stefanov and Anna Rita Giammetta and solves an open problem in the area and improves upon an earlier result in [1], where the authors have obtained scattering for the problem with nonlinearity $|u|^{p-1} u$, $p > 3$.

References

[1] Cuccagna S., Georgiev V., Visciglia N., Decay and scattering of small solutions of pure power NLS in R with p > 3 and with a potential, preprint, to appear CPAM.

◇Baoxiang Wang (Peking University)◇

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U2, V2-modulation spaces and nonlinear dispersive equations

We consider the Cauchy problem for the derivative NLS in 1D

$$i\partial_t u + \partial_x^2 u = i\mu\partial_x(|u|^2 u),\ \ u(x,0) = u_0(x)$$

It is known that DNLS is locally well posed in H^s for $s \geq 1/2$ and ill posed in H^s for $s < 1/2$. However, from the scaling argument we see that L^2 is a critical space in the sense that the scaling solution $u_\sigma(x,t) := \sigma^{1/2} u(\sigma x, \sigma^2 t)$ has an invariant norm in L^2 for all $\sigma > 0$. This fact implies that there is a gap between L^2 and $H^{1/2}$ for the well-posedness of DNLS. One can naturally ask what is the reasonable well-posed spaces with regularity below $H^{1/2}$ which can interpret its connections with L^2. We will consider the initial data in modulation spaces and show that DNLS is local well posed in $M^{1/2}_{2,p}$ with $2 \leq p < \infty$. There holds the following sharp embedding

$$M^{1/2}_{2,p} \subset B^{1/p}_{2,p} \subset B^0_{2\infty},\ \ 2 \leq p < \infty.$$

Our result contains all the subcritical modulation spaces $M^{1/2}_{2,p}$.

Similar result holds for the cubic NLS. If I have more time I will talk the related KdV equation.