International Workshop on the Multi-Phase Flow; Analysis, Modelling and Numerics

Waseda, November 10 – 13, 2015

Abstracts

TOP GLOBAL UNIVERSITY JAPAN

科研費 KAKENHI

早稲田大学 WASEDA University

Dieter Bothe

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Modeling and Direct Numerical Simulation of Transport Processes at Fluid Interfaces

The first main aim of this lecture series is to develop the continuum thermodynamical modeling of two-phase fluid systems, employing the sharp-interface approach. Starting with the detailed derivation of the two-phase Navier-Stokes system, we then consider liquid/vapor systems with phase change (evaporation, condensation) and, finally, extend the modeling to multicomponent two-phase systems with mass transfer and surfactants. Besides fundamental mathematical tools such as two-phase transport and divergence theorems, the main emphasis is on the entropy principle with surface contribution, allowing for a rational derivation of thermodynamically consistent jump and transmission conditions. The second main aim of this lecture series is to provide a brief overview of current numerical approaches for detailed simulations of such systems and to introduce some recent extensions in order to treat mass transfer multiphysics such as local volume changes or the effect of surface contamination.

Plan for the lectures:

Lecture 1: The two-phase Navier-Stokes equations. Sharp-interface continuum mechanical modeling.

Lecture 2: Liquid/vapour systems with phase change. Energy balance and the entropy principle.

Lecture 3: Multicomponent two-phase fluid systems. Mass transfer and surfactants.

Lecture 4: Numerical methods for transport processes at fluid interfaces. Mass transfer multiphysics.

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Marius Tucsnak

Institute of Mathematics of Bordeaux, University of Bordeaux, Bordeaux, France

Free and controlled particles in viscous flows

Lecture 1:

Rigid bodies immersed in a viscous incompressible fluid: basic modelling and analysis issues

We begin by introducing the PDE systems modelling the motion the motion of rigid bodies in a viscous fluid. We next describe the main difficulties encountered in the mathematical analysis of these problems and we recall the main existence and uniqueness of solutions results. Finally we discuss some issues related to low number flows in the incompressible case, with emphasis on the control theoretic interpretation of the swimming mechanism of aquatic microrganisms.

Lecture 2:
Analysis of the equations of a piston in viscous heat conducting gas

We consider a system coupling the one dimensional compressible Navier-Stokes-Fourier equations with an ODE coming from Newton's second law. This system, involving a free boundary, models the motion of a piston in a viscous heat conducting gas. We study the existence and uniqueness of global in time solutions. Moreover, we discuss the stabilizability of a toy model.

Lecture 3:
Motion of rigid bodies in a viscous fluid: flows in two or three space dimensions

We consider systems, modelling the motion of rigid bodies in a two or three dimensional viscous flow, which couple the Navier-Stokes equations with Newton's laws. The coupling is described by appropriate interface conditions and, since the domain filled by the fluid is not apriori known, we have to tackle a free boundary problem. We describe a methodology to obtain existence and uniqueness results, with emphasis on the case of incompressible fluids. In this second lecture we introduce the basic notions on control theory which will be necessary in the remaining part of these lectures. We consider both finite and infinite dimensional control systems and we recall some basic tools such as Chow's theorem for affine finite dimensional systems or duality of controllability and observability concepts.

Lecture 4:
Controlling the motion of solids immersed in a viscous fluid

In this lecture, after introducing some basic notions on control theory, we formulate two control problems for solids moving in a viscous incompressible fluid. In the first one, the aim consists in steering the bodies to prescribed positions by means of exterior forces acting on them. We show, in particular, that in the presence of control forces we can obtain existence and uniqueness results which are "better" than in the uncontrolled case. The second control mechanism consists in appropriate deformation of the solids: this is the swimming problem.

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Jan Dusek
Institute of Fluid and Solid Mechanics, University of Strasbourg, Strasbourg, France

Investigation of path instabilities of axisymmetric bodies falling or rising under the action of gravity and hydrodynamic forces in a Newtonian fluid using a spectral method

The non-linear modes of the bifurcation responsible for axisymmetry breaking in axisymmetric wakes are identical to modes of the azimuthal Fourier expansion making the latter an extremely efficient numerical discretization of 3D flows past axisymmetric bodies at the transition to turbulence. The efficiency of the method enables very extensive parametric studies of path instabilities of axisymmetric bodies of various shapes.

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Yasuhide Fukumoto
Institute of Mathematics for Industry, Kyushu University, Fukuoka, Japan

A higher-order asymptotic formula for traveling speed of a counter-rotating vortex pair

We establish a general formula for the traveling speed of a counter-rotating vortex pair, being valid for thick cores, moving in an incompressible fluid with and without viscosity. Two-dimensional motion of vortices with finite cores, interacting with each other, has been extensively studied both analytically and numerically. Mathematical methods and numerical schemes have been highly developed for dealing particularly with vortices of uniform vorticity, called vortex patches. In contrast, this is not the case with vortices with distributed vorticity. The method of matched asymptotic expansions was initiated by Ting and Tung (1965 Phys. Fluids Vol. 8, pp. 1039-1051) for this purpose.

We complete their program for a vortex pair, with a general vorticity distribution at the leading order, and derive a surprisingly simple formula for the correction, due to the effect of finite core size, to the traveling speed. The solution of the Navier-Stokes equations is constructed in the form of a power series in a small parameter, the ratio of the core radius to the distance between the core centers. A correction due to the effect of finite thickness of the vortices to the traveling speed makes its appearance at the 5th order. The 5th-order correction is shown to be expressible solely in terms of the 2nd-order quadrupole field.

An alternative route to reach the same formula is also sought. We devise a two-dimensional counterpart of Helmholtz-Lamb's formula which was originally developed for axisymmetric vortex rings.

(This is a joint work with Ummu Habibah)

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Giovanni Paolo Galdi
Department of Mechanical Engineering and Materials Science, Department of Mathematics, University of Pittsburgh, Pittsburgh, USA

On Bifurcating Time-Periodic Flow of a Navier-Stokes Liquid past a Cylinder

We provide general sufficient conditions for branching out of a time-periodic family of solutions from steady-state solutions to the two-dimensional Navier-Stokes equations in the exterior of a cylinder. To this end, we first show that the problem can be formulated as a coupled elliptic-parabolic nonlinear system in appropriate function spaces. This is obtained by separating the time-independent averaged component of the velocity field from its "purely periodic" one. We then prove that time-periodic bifurcation occurs, provided the linearized time-independent operator of the parabolic problem possess a simple eigenvalue that crosses the imaginary axis when the Reynolds number passes through a (suitably defined) critical value. This assumption is validated by well-known numerical tests. We also show that only supercritical or subcritical bifurcation may occur.

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Roland Glowinski
Department of Mathematics, University of Houston, Huston, USA

A Distributed Lagrange Multiplier/Fictitious Domain/Immersed Boundary method for simulating compound vesicle motion under creeping flow condition

In this lecture we present first a distributed Lagrange multiplier/fictitious domain (DLM/FD) method for simulating fluid-particle interaction in Stokes flow. A conjugate gradient method driven by both pressure and distributed Lagrange multiplier, called one-shot method, has been developed to solve the discrete Stokes problem while enforcing the rigid body motion within the region occupied by the particle. The methodology is validated by comparing the numerical results of a neutrally buoyant particle of either a circular or elliptic shape with the associated Jeffery's solutions. We have successively combined the above methodology with an immersed boundary (IB) method and an elastic membrane modeled by a spring network to simulate the dynamics of a compound vesicle. In simple shear flow under creeping flow condition, the results are consistent with those obtained in literature. In Poiseuille flow, the compound vesicle motion is dominated by the motion of the vesicle membrane as expected and stays in the central region of the channel.

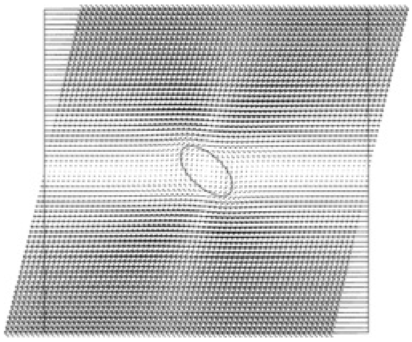

Figure 1: Snapshot of the velocity field near to an elliptic shape particle

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Yoshiyuki Kagei

Department of Mathematical Sciences, Kyushu University, Fukuoka, Japan

Traveling waves bifurcating from Poiseuille flow in viscous compressible fluid

In this talk I consider the stability of Poiseuille flow of the compressible Navier-Stokes equation in an infinite layer. I will give an asymptotic description of large time behavior of solutions around Poiseuille flow when the Reynolds and Mach numbers are suficeintly small. Instability of Poiseuille fow will then be considered. A condition for the Reynolds and Mach numbers will be given in order for Poiseuille flow to be unstable. It will be shown that plane Poiseuille flow is unstable for Reynolds numbers much less than the critical Reynolds number for the incompressible flow when the Mach number is suitably large. We will finally discuss bifurcation of traveling waves from Poiseuille flow. This talk is based on a joint work with Takaaki Nishida (Kyoto University).

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Shuichi Kawashima

Department of Mathematical Sciences, Kyushu University, Fukuoka, Japan

Mathematical entropy and Euler-Maxwell system

The notion of the mathematical entropy was first introduced by Godunov (in 1961) for hyperbolic systems of conservation laws. The notion was then extended by Kawashima-Shizuta (in 1988) for hyperbilic-parabolic systems of conservation laws and by Kawashima-Yong [1] (in 2004) for hyperbolic systems of balance laws with symmetric relaxation. In this talk, we modify the definition of the mathematical entropy in [1] so that it is valid for systems with non-symmetric relaxation. Also we discuss the Euler-Maxwell system as an example of hyperbolic balance laws with non-symmetric relaxation.

References

[1] S. Kawashima and W.-A. Yong, Dissipative structure and entropy for hyperbolic systems of balance laws, Arch. Rat. Mech. Anal., **174** (2004), 345–364.

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Takayuki Kobayashi
Department of Systems Innovation, Osaka University, Osaka, Japan

Decay estimates of the solutions to the 2D linear viscoelastic equation and its applications to the compressible Navier-Stokes equation

We consider the Cauchy problem of 2D linear viscoelastic equation and show the decay property for the solutions in the L2 framework. The linear viscoelastic equation is the linearlized equation of the density part in the compressible Navier-Stokes equation around a given constant equilibrium. We will observe that there appear some aspects different from the time decay rate in space L2-norm of the solutions, especially in considering space-time L2-norm of the solutions. The results in this talk were obtained in a joint work with M. Misawa (Kumamoto University, Japan) and T. Yanagisawa (Nara Women's University, Japan).

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Seiichi Koshizuka
Department of Systems Innovation, The University of Tokyo, Tokyo, Japan

Particle Method: a Lagrangian Meshless Method for Computational Fluid Dynamics

Particle method is Lagrangian meshless approach in computational fluid dynamics. Numerical diffusion derived from the discretization of the convection terms does not arise due to Lagrangian description. We do not need to care grid deformation in complex motion of fluid interfaces. Basic concept, algorithms and application to various problems are presented.

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Takayuki Kubo
Faculty of Pure and Applied Sciences, University of Tsukuba, Tsukuba, Japan

On some two phase problem for compressible-compressible viscous fluid flow

In this talk, we consider some two phase problem for compressible-compressible viscous fluid flow. In order to prove local well-posedness and global well-posedness for our problem, the generation of analytic semigroup for linearlized problem and its maximal $L_p - L_q$ regularity theorem are needed in our method. The key step to prove them is to prove the existence of $\mathcal{R}$-bounded solution operator to the resolvent problem corresponding the linearized problem. In this talk, we shall report the existence of $\mathcal{R}$-bounded solution operator for the resolvent problem. Moreover as its application, we shall introduce the local well-posedness and global well-posedness for our problem.

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Takashi Kumagai

Research Institute for Mathematical Sciences, Kyoto University, Kyoto, Japan

Anomalous random walks and diffusions in random media

In this talk, I will survey recent developments on anomalous (and reversible) random walks/diffusions in random media.

I will mainly focus on the random conductance model, in which conductances are given randomly on each bond in the d-dimensional lattice. I will discuss detailed heat kernel estimates and scaling limits of random walks for the random conductance model. I will explain why some of these random walks/diffusions behave anomalously.

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Tetu Makino

Yamaguchi University, Yamaguchi, Japan

On the Einstein-Euler equations

Spherically symmetric solutions to the Einstein-Euler equations with physical vacuum boundary can be constructed near time periodic linearized approximations around equilibrium governed by the Tolman-Oppenheimer-Volkoff equations, provided that the material perfect fluid obeys an approximate γ-law with $\gamma/(\gamma - 1)$ being an integer. The Cauchy problem can be solved, too. But, in contrast with the non-relativistic problem governed by the Euler-Poisson equations, the extension of the metric into the exterior vacuum region is not trivial. The region of positive density and that of vacuum should be considered as those occupied by two phases of the matter.

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Yoichiro Matsumoto
Riken, Saitama, Japan

Mathematical Modeling of Acoustic Response of Bubble Cloud

Acoustic response of bubble cloud is one of the most interesting problems in fluid mechanics. The behavior is governed by single bubble dynamics and also close interaction among bubbles. We discuss single bubble motion, pressure wave propagation in bubbly flow and acoustic response of bubble cloud. Single bubble motion: The bubble motion in an acoustic field is observed by the experiment for comparison with a numerical simulation. The calculation is based on a mathematical model in which the thermo-fluid dynamics of the gas in the bubble is precisely described. An oscillatory pressure field is generated in a cylindrical cell, which consists of two piezo ceramic transducers and a glass cylinder. Air bubble is injected in the acoustic field. The bubble motion is observed by high-speed photography. A quantitatively good agreement between the experimental and numerical results is achieved through the precise description of the thermal behavior inside a bubble, which indicates the importance of the internal thermal phenomena. Pressure wave propagation in bubbly flow: Numerical and experimental studies of the transient shock wave phenomena in a liquid containing non-condensable gas bubbles are conducted. In the numerical analysis, individual bubbles are tracked to estimate the effect of volume oscillations on the wave phenomena. Thermal processes inside each bubble are calculated directly using full equations for mass, momentum and energy conservation, and those results are combined with the averaged conservation equations of the bubbly mixture to simulate the propagation of the shock wave. Experiments are done in a shock tube with an upwardly directed bubbly flow to obtain a uniform spatial distribution of bubbles. The transient pressure profiles obtained in the experiment for the upward bubbly flow agree well quantitatively with those obtained by the numerical calculation using a uniform spatial distribution of bubbles. The results reveal that the bubble distribution and each bubble behavior have important effects to the characteristics of the shock wave propagation. Acoustic response of bubble cloud: Strong pressure peak is generated at the center area in a cloud cavitation. As these dynamics of bubbles are strongly influenced by the thermal phenomena inside them, it is necessary to construct the model taking these phenomena into account to analyze the behavior of the bubble cloud precisely. When the frequency of the ultrasound is sufficiently high, the bubble cloud hardly oscillates. On the contrary, when the frequency of the ultrasound is at the resonance of the bubble cluster, the pressure wave generates the shock wave and it focuses to the cluster center. As a result, the pressure inside the bubble at the cluster center becomes much higher than that of a single bubble. Though this extreme high pressure causes the severe cavitation erosion, it is thought that this high energy concentration has the potential to be utilized for various applications. As one of the examples related to the bubble cloud behaviors, microbubble clusters consisted of ultrasound contrast agent are experimentally investigated. The maximum size of the clusters to be captured by focused ultrasound is discussed through the comparison with a classical theory for a bubble cloud. Although the ultrasound contrast agent microbubbles used in the present experiments have a surface coated by lipid molecules, the dependence of the maximum cluster size on the ultrasound frequency shows qualitative agreement with the theory.

(This is a joint work with Masaharu Kameda, Shin Yoshizawa and Shu Takagi)

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Hirofumi Notsu
Waseda Institute for Advanced Study, Waseda University, Tokyo, Japan

Error estimates of Lagrange-Galerkin schemes for flow problems

Stable and stabilized Lagrange-Galerkin (LG) schemes for the Navier-Stokes and natural convection problems are considered. Optimal error estimates of a stable LG scheme for the Navier-Stokes problems have been proved in [Süli, Numer. Math., 53 (1988), 459-483]. We present optimal error estimates of a stabilized LG for the Navier-Stokes problems and stable and stabilized LG schemes for natural convection problems.

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Marta Sanz-Solé
Faculty of Mathematics, University of Barcelona, Spain

Hitting probabilities for solutions of SPDEs

We will discuss a methodology to obtain upper and lower bounds for hitting probabilities of random fields. This is motivated by the study of polar sets, level sets and sections of solutions to systems of stochastic partial differential equations. Our main focus will be on the stochastic wave equation in spatial dimensions $k \leq 3$. A crucial challenge is to obtain the rate of degeneracy of L^p moments of the inverses of the eigenvalues of the Malliavin matrix of vectors of solutions at two different points, when they go close to each other. The influence of that rate over the optimality of the lower bound for the hitting probabilities will be highlighted. This is joint work with R. Dalang (EPFL, Switzerland).

In the last part of the talk, we will outline some preliminary results on a stochastic Poisson equation driven by an additive Gaussian noise. The study starts with the Gaussian case and then it is extended to some class of semilinear elliptic systems, using Kusuoka's extension of Girsanov's theorem. This is ongoing work with N. Viles (UB, Spain).

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Kenji Takizawa
Department of Modern Mechanical Engineering, Waseda University, Tokyo, Japan

Computational engineering analysis with the new-generation space-time methods

In this presentation, an overview of the new directions in the space-time (ST) methods will be explained. The classes of problems our team have focused on include spacecraft parachute, bio-inspired flapping-wing aerodynamics, turbomachinery, and cardiovascular fluid mechanics. The new directions for the ST methods include the variational multiscale version of the Deforming-Spatial-Domain/Stabilized ST method, using NURBS basis functions in temporal representation of the unknown variables and motion of the solid surfaces and fluid meshes, ST techniques with continuous representation in time, and ST interface-tracking with topology change.

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Noboru Chikami
Mathematical Institute, Tohoku University, Sendai, Japan

Global solution for the Navier-Stokes-Poisson system in two and higher dimension

This talk is based on a joint work with R. Danchin (UPEC). We consider the Cauchy problem of the compressible Navier-Stokes equations with a Poisson-type potential in the whole space:

$$\begin{cases} \partial_t\rho + \mathrm{div}\,(\rho u) = 0, & (t,x) \in \mathbb{R}_+ \times \mathbb{R}^n, \\ \partial_t(\rho u) + \mathrm{div}\,(\rho u \otimes u) + \nabla P(\rho) & \\ \qquad = \mu\Delta u + (\lambda+\mu)\nabla\mathrm{div}\, u + \kappa\rho\nabla\psi, & (t,x) \in \mathbb{R}_+ \times \mathbb{R}^n, \\ -\Delta\psi = \rho - \underline{\rho}, & (t,x) \in \mathbb{R}_+ \times \mathbb{R}^n, \\ (\rho, u)|_{t=0} = (\rho_0, u_0), & x \in \mathbb{R}^n, \end{cases} \tag{0.1}$$

where $\rho = \rho(t,x)$, $u = u(t,x)$ and $\psi = \psi(t,x)$ are the unknown functions, representing the fluid density, the velocity vector and the potential force, respectively. The pressure $P = P(\rho)$ is given by a smooth function only depending on ρ and we assume $\mu > 0$, $\lambda + 2\mu > 0$ and $\underline{\rho} > 0$.

We obtain a new a priori estimate of the solutions for the system (0.1) to show a unique global solvability in two and higher dimensions in a low regularity Besov framework. To our knowledge, all previous works excluded the two-dimensional case. The key ideas of the proof are a very simple L^2-type energy estimate and an introduction of new unknown called an *effective velocity*, first used in the context of low-regularity well-posedness by P.L. Lions and B. Haspot.

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Shota Enomoto
Department of Mathematical Sciences, Kyushu University, Fukuoka, Japan

On asymptotic behavior of spatially periodic stationary solutions to the compressible Navier-Stokes equation in a periodic layer

We consider large time behavior of solution to compressible Navier-Stokes equation around the spatially periodic stationary solutions in a periodic layer of R^n $(n = 2, 3)$.There exists a spatially periodic stationary solution if the external force is spatially periodic and is sufficiently small in some Sobolev space.We show that the L^2 norm of perturbation decays in the order $t^{-\frac{n-1}{4}}$ as $t \to \infty$ if the initial perturbation is sufficiently small; in fact, the perturbation behaves diffusively in large time.

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Miho Murata
Department of Mathematics, Waseda University, Tokyo, Japan

Fluid-rigid body interaction problem for compressible fluids

We consider the system of equations describing the motion of a rigid body immersed in a compressible viscous fluid within the barotropic regime. The main result of this talk states that we have the existence and uniqueness of local strong solution in the L_p in time and L_q in space setting with $2 < p < \infty$ and $3 < q < \infty$. For the purpose, we use the contraction mapping principle based on the maximal L_p-L_q regularity. Concerning the same coupled system, Boulakia and Guerrero [1] proved an existence result for strong solution in the L_2 setting. One of the merits of our approach is less compatibility condition and regularity on initial data compared with the ones given in [1].

References

[1] M. Boulakia and S. Guerrero, A regularity result for a solid-fluid system associated to the compressible Navier-Stokes equations, *Ann. Inst. H. Poincaré Anal. Non Linéaire* **26** (2009), 777–813.

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Issei Oikawa
Department of Mathematics, Waseda University, Tokyo, Japan

HDG methods with reduced stabilization

In this talk, we present hybridized discontinuous Galerkin(HDG) methods with reduced stabilization. The reduced stabilization enables us to use polynomials of degree $k + 1$ and k to approximate element and hybrid unknowns, respectively, which improves the convergence of the methods. Error analysis and numerical results are also presented.

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Linyu Peng
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Formal variational structures and geometric integrators: A first attempt to fluid dynamics

For differential equations governed by variational principles, not only we have the celebrating Noether's theorem connecting symmetries and conservation laws but also often the conservation of symplecticity or multisymplecticity is achievable. For many cases, however, promised by the inverse theory no variational principles would exist. By introducing formal Lagrangians, adjointness theory secures us from the lack of variational principles and provides fruitful results, especially for constructing conservation laws from symmetries of differential equations. In this talk, we will generalise these ideas to suit fluid dynamics and hope to shed new light on geometric integrators for fluid dynamics.

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Kazuyuki Tsuda

Graduate School of Mathematics, Kyushu University, Fukuoka, Japan

Time periodic problem for the compressible Navier-Stokes-Korteweg system on the whole space

We consider time periodic problem for the compressible Navier-Stokes-Korteweg system (CNSK) on $\mathbb{R}^3$. It is known that CNSK is a model system for two phase flow with phase transition between liquid and vapor in compressible fluid. In this model we consider the phase transition boundary as a diffuse interface. CNSK is the system adding the Kortweg tensor to the compressible Navier-Stokes equation.

Cai, Tan and Xu ([1]) considered the time periodic problem for CNSK on $\mathbb{R}^3$ with $n \geq 5$. They proved that there exists a time periodic solution around the motionless state for a sufficiently small time periodic external force. Furthermore, they showed that the time periodic solution is stable under sufficiently small perturbations and the L^∞ norm of the perturbation decays as time goes to infinity. In this talk, we consider CNSK on $\mathbb{R}^3$. We show the existence of a time periodic solution for the sufficiently small external force. It is also shown that we have the stability of the time periodic solution under sufficiently small perturbations and the decay of L^∞ norm of the perturbation.

References

[1] H. Cai, Z. Tan and Q. Xu, Time periodic solutions of the non-isentropic compressible fluid models of Korteweg type, Kinet. Relat. Models., **8** (2015), pp. 29–51.

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Kenkichi Tsunoda

Graduate School of Mathematical Sciences, The University of Tokyo, Tokyo, Japan

Derivation of Stefan problem from a one-dimensional exclusion process with speed change

Bertsch, Dal Passo and Mimura [1] introduced a mathematical model which describes a phenomenon of two different types of cells, called “contact inhibition of growth between two cells”. They formulated this phenomenon as a one-dimensional system of partial differential equations. In some cases, the system of partial differential equations can be expressed as a Stefan free boundary problem. Our goal is to relate a microscopic particle system to the macroscopic Stefan free boundary problem. We study a model of a microscopic particle system called an exclusion process with speed change to derive the Stefan free boundary problem. From the point of view of the particle system, a solution to the partial differential equation describes an evolution of the macroscopic density of particles and it is derived under a diffusive scaling limit called the hydrodynamic limit. On the other hand, the moving Stefan free boundary corresponds to the behavior of a tagged particle in the particle system. The hydrodynamic limit for the exclusion process with speed change was already studied. In this talk, we see the behavior of a tagged particle for the particle system and a law of large numbers for a tagged particle under the diffusive scaling. Combining this with the result of the hydrodynamic limit, we derive a Stefan free boundary problem in a diffusive scaling limit.

References

[1] M. Bertsch, R. Dal Passo, M. Mimura, *A free boundary problem arising in a simplified tumor growth model of contact inhibition*, Interfaces and Free Boundaries, **12** (2010), 235-250.

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Satoshi Yokoyama

Graduate School of Mathematical Sciences, The University of Tokyo, Tokyo, Japan

Existence and uniqueness of solutions to stochastic Rayleigh-Plesset equations

In this talk, we consider Rayleigh-Plesset equations perturbed by random forces. The Rayleigh-Plesset equation, which is an ODE, is known as that which describes the motion of the radial part of a single spherical gas bubble moving in an incompressible flow. However, the existence and uniqueness of its solution have not been discussed. Our purpose is to formulate an SDE by adding random forces which are considered to be natural in some ways from the physical viewpoint. As such random forces, we introduce noises to the velocity field and the boundary of the gas bubble. Under these settings, by means of a suitable Lyapunov function and Itô's formula, we prove that the unique solution of the SDE does not blow up. Furthermore, the existence of the associated invariant measure is shown.

References

[1] T. Funaki, M. Ohnawa, Y. Suzuki, S. Yokoyama, *Existence and uniqueness of solutions to stochastic Rayleigh-Plesset equations*, J. Math. Anal. Appl. **425** (2015), 20–32.

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