

**“Multiscale Analysis, Modeling and Simulation”**  
**Top Global University Project, Waseda University**  
**REPORT ON STUDY ABROAD**

Date: August 15, 2015

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- 1. Study abroad destination: TU Darmstadt, Germany**
- 2. Dates of stay: July 6, 2015 – August 6, 2015 (30days)**
- 3. Purpose:** To develop the research on weak and strong solutions to the Navier-Stokes equations and study the mathematical theory of fluid dynamics at TU Darmstadt.
- 4. Host Professor: Prof. Reinhard Farwig (TU Darmstadt)**
- 5. Education and research activity in the destination**
  - I. Seminars, lectures, Conferences, etc:**
    - (1) Patrick Tolksdorf (TU Darmstadt), “Gradient estimates for the Stokes resolvent in bounded Lipschitz domains”, IRTG seminar, TU Darmstadt (Germany) July 07, 2015.
    - (2) Gisele Goldstein (University of Memphis), “The linear PDEs of mathematical finance” IRTG seminar, TU Darmstadt (Germany) July 14, 2015.
    - (3) Jerry Goldstein (University of Memphis), “Energy asymptotics of non autonomous waves” IRTG seminar, TU Darmstadt (Germany) July 14, 2015.
    - (4) Roland Schnaubelt (Karlsruhe), “A structurally damped plate equation with Dirichlet-Neumann boundary condition” TULKKA-Treffen in Ulm, ulm university (Germany) July 17, 2015.
    - (5) Jerome Goldstein (University of Memphis), “The PDEs of mathematical finance” TULKKA-Treffen in Ulm, ulm university (Germany) July 17, 2015.
    - (6) Marco Ritter (Konstanz), “On a thermoelastic system in exterior domains” TULKKA-Treffen in Ulm, ulm university (Germany) July 17, 2015.
    - (7) Martin Adler (Tübingen), “Störungstheorie für Generatoren stark stetiger Halbgruppen” TULKKA-Treffen in Ulm, ulm university (Germany) July 17, 2015.
    - (8) Matthias Hieber (Darmstadt), “Periodic Solutions to Linear and Semilinear Evolution Equations” TULKKA-Treffen in Ulm, ulm university (Germany) July 17, 2015.

## **II. Presentations :**

- (1) Partial Regularity and Extension of Solutions to the Navier-Stokes Equations, IRTG seminar, TU Darmstadt (Germany) July 07, 2015.

## **III. Research Results:**

I have been investigating the mathematical theory of incompressible Navier-Stokes equations since I entered the Graduate school of Waseda University. Particularly, I have taken a great interest in the smoothness of suitable weak solutions and time-extension criteria for local-in-time solutions. It is a well-known fact that this Cauchy problem for the incompressible Navier-Stokes equations is time locally well-posed, therefore it is an important work to establish the time-extension criterion for local-in-time solutions. By utilizing some techniques that have been used in the partial regularity theory for suitable weak solutions, I have proved one time-extension criterion which means that local-in-time solutions can be continued beyond the final time if some local Morrey-type functional of the solutions is sufficiently small near the final time. Furthermore, I submitted one paper on this topic.

During my research stay at Technical University Darmstadt, I improved my results and established the time-extension criterion for local-in-time solutions by estimating the Morrey type functional when the domain is smoothly bounded domain. Since my previous result focused on only Cauchy problem, by using much deeper argument for the partial regularity I have got this relaxation. There are still some difficulties in the case of another domain such as exterior domain because of the structure of pressure and decomposition of the pressure. Moreover, the lively discussions with Prof. Farwig made me find some interesting future works. This is about the new regularity theorem for suitable weak solutions. Seregin-Sverak(2002) established a regularity theorem for suitable weak solutions by estimating a Morrey type functional, and then proved that weak solutions are smooth and unique if the pressure is bounded from below. First of all, I will improve this regularity theorem by estimating a Campanato type functional instead of estimating the Morrey type functional. Then, I will try to prove another time-extension criterion for local-in-time solutions by estimating some Campanato type functional. Moreover, I really want to improve the Seregin-Sverak's result about the pressure condition which is strongly related to the phenomenon of cavitation, more precisely the aim here is to get the smoothness of solutions under more relaxed condition on the pressure.

## **6. Other comments:**

These 30 days were very busy and eventful. I gave a talk at the seminar in Technical University Darmstadt the day after my arrival in Germany. Although it was a bit hard schedule for me, I got several nice questions and comments and they inspired me. The following week I attended the PhD Defenses by two students, who had studied fluid mathematics at Technical University Darmstadt. It was not easy for me to comprehend everything because their talk were in German, but I understood what they proved mathematically and learned a lot about how to behave during my PhD defense in the future. After the defenses other German students and I celebrated them and held a huge party. Thanks to Prof. Hieber, I got a chance to attend the conference in Ulm University around the middle of my stay. It was a good experience that I got acquainted with some German students from other universities, such as the University of Konstanz.

All German students are really getting along well with each other, and I spent so nice time with them. Some students kindly invited me to their apartments and cooked me delicious German foods for dinner. We had great times talking about the cultural difference between Japan and Germany, international affairs, diplomacy, the security bill in Japan, and of course sometimes mathematical problems in fluid mathematics. The interaction with German students was definitely invaluable and the discussion on mathematics with them and the professors have been powerful incentives for me to work on my research much harder.

Darmstadt is very safe city and the environment for researchers in Technical University Darmstadt is so perfect that I led a full research life there. I am of the firm conviction that Japanese students will gain precious experience and enjoy their lives if they come to Darmstadt to study mathematics.

Lastly, I would like to express my deepest gratitude to Prof. Shibata and this project itself for giving me such a nice opportunity to study abroad for one month. I really hope much more Japanese students study abroad and learn a lot by experience throughout this program.