“ Multiscale Analysis, Modeling and Simulation”

-Top Global University Project, Waseda University-

REPORT ON STUDY ABROAD

Date: March 23, 2015

Name: Shoh Takahashi

1. Study abroad destination: Technische Universität Darmstadt,Germany
2. Dates of stay: February 10, 2015 March 9, 2015 (28 days)
3. Purpose: To become mature person who can active with global view through proceeding the study of the linearized MHD equation by discussing with Prof. Dr. Matthias Hieber.
4. Host Professor: Prof. Dr. Matthias Hieber (Technische Universität Darmstadt)
5. Education and research activity in the destination: I proved the following (SEMI) and (ES) in this study abroad.
   (a) At first, I came to the conculusion about $\mathcal{R}$-boundedness for the magnetic field $\mathbf{H}$ on (MHD) with initial data $(\mathbf{F}_+,\ \mathbf{F}_-) \in X_q$. Next, we led the resolvent estimates by $\mathcal{R}$-boundedness. Finally, we constructed the semigroup $\{\mathrm{e}^{-\mathcal{A}t}\}_{t\geq 0}$ on X_q with $1 < q \leq r \leq \infty$, $t \geq 0$ and got the folloing estimates (SEMI).

$$\begin{cases} \mu_+\partial_t \mathbf{H}_+ + \sigma^{-1}\,\mathrm{rot}\,\mathrm{rot}\,\mathbf{H}_+ = 0, \quad \mathrm{div}\,\mathbf{H}_+ = 0 & \text{in } \mathbf{R}^3_+, \\ \mathrm{rot}\,\mathbf{H}_- = 0, \quad \mathrm{div}\,\mathbf{H}_- = 0 & \text{in } \mathbf{R}^3_-, \\ [\![\mathbf{H}_\tau]\!] = 0, \quad [\![\mu\mathbf{H}\cdot\mathbf{n}]\!] = 0 & \text{on } \mathbf{R}^3_0, \\ \mathbf{H}|_{t=0} = \mathbf{F} & \text{in } \dot{\mathbf{R}}^3. \end{cases} \tag{MHD}$$

$$\begin{aligned} H_q(\mathbf{R}^3_-) &= \{\mathbf{u} \in L_q(\mathbf{R}^3_-)^3 \mid \mathrm{div}\,\mathbf{u} \in L_q(\mathbf{R}^3_-)\}, \\ R_q(\mathbf{R}^3_-) &= \{\mathbf{u} \in L_q(\mathbf{R}^3_-)^3 \mid \mathrm{rot}\,\mathbf{u} \in L_q(\mathbf{R}^3_-)^3\}, \\ X_q :&= \{(\mathbf{F}_+\ \mathbf{F}_-) \mid \mathbf{F}_+ \in L_q(\mathbf{R}^3_+)^3,\ \mathbf{F}_- \in H_q(\mathbf{R}^3_-) \cap R_q(\mathbf{R}^3_-),\ \mathrm{div}\,\mathbf{F}_- = 0, \\ &\qquad \mathrm{rot}\,\mathbf{F}_- = 0 \text{ in } \mathbf{R}^3_-,\ (\mu\mathbf{F}, \nabla\psi)_{\dot{\mathbf{R}}^3} = 0 \text{ for all } \psi \in W^1_{q'}(\dot{\mathbf{R}}^3)\}. \end{aligned}$$

$$\begin{cases} \|\nabla^l \mathrm{e}^{-\mathcal{A}t}\mathbf{F}\|_{L_r(\mathbf{R}^3_+)} \leq Ct^{-\frac{3}{2}(\frac{1}{q}-\frac{1}{r})-\frac{l}{2}}\|\mathbf{F}\|_{X_q} \quad (l = 0, 1), \\ \|\nabla^2 \mathrm{e}^{-\mathcal{A}t}\mathbf{F}\|_{L_r(\mathbf{R}^3_+)} \leq Ct^{-\frac{3}{2}(\frac{1}{q}-\frac{1}{r})-1}\|\mathbf{F}\|_{X_q}. \quad (l = 2,\ r \neq \infty). \end{cases} \tag{SEMI}$$

(b) I estimated a unique solution $\mathbf{H}$ on (MHD2) with a force power $\mathbf{G}$ by $L_p - L_q$ maximum regularity and got the folloing estimate (ES).

$$\begin{cases} \mu_+\partial_t\mathbf{H}_+ + \sigma^{-1}\,\mathrm{rot}\,\mathrm{rot}\,\mathbf{H}_+ = \mathbf{G}, \quad \mathrm{div}\,\mathbf{H}_+ = 0 & \text{in } \mathbf{R}^3_+, \\ \mathrm{rot}\,\mathbf{H}_- = 0, \quad \mathrm{div}\,\mathbf{H}_- = 0 & \text{in } \mathbf{R}^3_-, \\ [\![\mathbf{H}_\tau]\!] = 0, \quad [\![\mu\mathbf{H}\cdot\mathbf{n}]\!] = 0 & \text{on } \mathbf{R}^3_0, \\ \mathbf{H}|_{t=0} = 0 & \text{in } \dot{\mathbf{R}}^3. \end{cases} \tag{MHD2}$$

$$\|\mathrm{e}^{-\gamma t}(\partial_t\mathbf{H},\ \Lambda_\gamma^{\frac{1}{2}}\nabla\mathbf{H},\ \nabla^2\mathbf{H})\|_{L_p(\mathbf{R},\ L_q(\mathbf{R}^3))} \leq C_{p,q}\|\mathrm{e}^{-\gamma t}\mathbf{G}\|_{L_p(\mathbf{R},\ L_q(\mathbf{R}^3_+))}. \tag{ES}$$

According to (a), I proved $L_q - L_r$ decay estimates for the magnet part on the original MHD(Magnetohydrodynamics) equation. I puzzled the following things (1) Which functional space is appropriate for "X_q"? (2) I need to modify my master thesis by $\mathcal{R}$-boundedness in terms of transforming a linerlized MHD equation into a nonlinerlized one.

Especially, X_q is one of the most important parts in my master thesis because it lets an auxiliary resolvent equation return an original resolvent equation on (MHD). I had trial and error and got somewhat effective X_q when I was in Japan. However, it was not perfect. I could get an appropriate X_q which led my study to the goal in Germany.

6. Impression for this program/ Advantage for my future:

I could proceed my master thesis through this program in Germany very well. That was due to having enough time to be absorbed in study, getting relax through the fine weather and many histrical cities, and the influence of the Germans' agressive actions and their deep knowledge for the study.

I will start to work in financial area from this April and I'm required to go abroad to work within the next first 5 years, at least, the term will be over 1 year. Both to communicate with a lot of Germans without Japanese and to live independently in this medium and long term in Gernany will have to be really mental nourishment to active globally in the future.

In a nutshell, I could gain a lot of knowledge and priceless experience through " Top Global University Project".