

**“Multiscale Analysis, Modeling and Simulation”
Top Global University Project, Waseda University
REPORT ON STUDY ABROAD**

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- 1. Study abroad destination: Technical University Darmstadt, Germany**
- 2. Dates of stay: February 10, 2015 – March 9, 2015 (28days)**
- 3. Purpose:** To study a mathematical fluid dynamics in Technical University Darmstadt, Germany
- 4. Host Professor: Prof. Matthias Hieber (TU Darmstadt)**
- 5. Education and research activity in the destination:**

I first in this stay had a discussion about a *two-phase* free boundary problem of the incompressible Navier-Stokes equations for a class of non-Newtonian fluids with Prof. Matthias Hieber. The discussion was concerned with some extension of our joint work, which had proved the local well-posedness of the two-phase free boundary problem and had been obtained in the last research stay at TU Darmstadt. More precisely, we considered how to show the global well-posedness, and we decided to start with a Newtonian situation instead of the non-Newtonian situation. I suggested that an approach due to Saito and Shibata (2015) would be useful in analysis of the Newtonian case.

Next I introduce some results, obtained in this research stay, about a *one-phase* free boundary problem of the incompressible Navier-Stokes equations in the case of Newtonian fluids. The one-phase free boundary problem is the so-called *ocean problem* without gravity or surface tension, that is, the Navier-Stokes equations in the problem describe flows of an incompressible and viscous fluid bounded above by a free surface and below by a solid surface.

As the first step, I showed the maximal L_p - L_q regularity with exponential stability for some linearized problem of the ocean problem. Roughly speaking, I proved the property as follows: Saito (2015) furnished the maximal L_p - L_q regularity with exponential stability for some time-shifted problem, while Abels (2006) an analytic semi-group, exponential stable, associated with the linearized problem. Then an extra term in the

time-shifted problem could be estimated by using the analytic semi-group, so that the required property was true.

As the second step, I considered a nonlinear problem on some fixed domain, which had been obtained by using Lagrangian transformation. We then used the maximal L_p - L_q regularity theorem obtained in the first step and contraction mapping theorem, and thus we proved the global well-posedness of the nonlinear problem on the fixed domain. In addition, the solution constructed in that way was exponentially stable.

As the third step, I obtained a unique solution to the ocean problem by applying the inverse Lagrangian transform to the solution of the nonlinear problem on the fixed domain .

In the last part of the research stay, I discussed with Jonas Sauer who is a Ph.D. course student of TU Darmstadt. One of his researches was concerned with the Stokes resolvent equations on periodic domains with respect to the first variable and in weighted function spaces. He proved resolvent estimates of parabolic type under the situation. To show the estimates, he first decomposed solutions of the Stokes resolvent equations into several parts represented by solutions of the Poisson equations with appropriate boundary conditions. Next he established some methods to analyze the Poisson equations under the same situation as the Stokes resolvent equations. Finally the above argumentations combined furnished the required resolvent estimates.

6. Other comments:

Good points of the research stay were the followings: First one was to communicate with young mathematicians. It stimulated me quite a lot, and especially the discussion with Jonas Sauer was very interesting and exciting. Second one was to concentrate my own research as mentioned above. Third one was to communicate in English with each other. The situation improved my English level to discuss with foreign researches in English smoothly.

In view of my experiences, I believe that Top Global University Project will produce talented people being active in the world.

Finally I am greatly indebted to Prof. Yoshihiro Shibata for giving me an opportunity to study abroad.