Research Report（September, 2020- September, 2021）

Enrollment from September 2020 | Department of Pure and Applied Mathematics | Fumitaka WAKABAYASHI

I. List of Papers

II. List of Talks

1. Removability of time-dependent singularities in the Stokes equations, The Mathematical Society of Japan, Keio University (Online), March 2021
2. Removability of time-dependent singularities of the Navier-Stokes equations, The Mathematical Society of Japan, Chiba University (Online), September 2021

III. Research Results in AY2020

We extended the removability of time-dependent singularities of the Stokes equations to that of the Navier-Stokes equations in $\Omega \times (0,T)$(Ω is bounded.). Specifically, we showed that the α-Holder continuous singularity $\xi(t)$ of the solution u is removable if the singularity of u at $x = \xi(t)$ is weaker than that of $|x|^{-1}$ and $|x|^{1/\alpha - N}$, that is, there exists the solution $\bar{u}$ in $\Omega \times (0,T)$ such that $\bar{u} \equiv u$ in $\Omega \times (0,T)$ except for $x = \xi(t)$. Furthermore, we tried to use this method to the 3D Keller-Segel system of parabolic-elliptic type, but we failed to obtain the removability of time-dependent singularities since it is difficult to estimate the gradient of test functions under the assumption that the solution belongs the scale invariant space $L^{3/2,\infty}$.

We also consider the solution with time-dependent singularities to the Navier-Stokes equations. The first is to guarantee the smoothness of the solution constructed by Karch-Zheng. It seemed to be available to make use of the method that considers the Stokes equations with the perturbed term whose coefficient is the solution to the Navier-Stokes equations and the uniqueness of the perturbed Stokes equations. But that is difficult because of the bad boundary condition. The second is to construct the solution by giving the external force with time-dependent singularities. By the existence theorem of the solution to the Navier-Stokes equations in the Bezov space, we constructed the solution by giving the time-dependent delta function in 2 dimensions and the single layer potential whose support is on the time-dependent sphere in 3 dimensions as the external force.

IV. Research Plan for AY2021

The first goal is to improve the result of Karch-Zheng in the Navier-Stokes equations. In 3 dimensions, they constructed singular distributional solutions if the singularity is α-Holder continuous $(1/2 < \alpha \leq 3/4)$ in time, and smooth singular solutions if the singularity is α-Holder continuous $(3/4 < \alpha)$ in time. It seems to be natural that we can also construct the smooth singular solutions when $1/2 < \alpha \leq 3/4$ since from our result we see that the singularity $\xi(t)$ of the solution is removable if the singularity of the solution at $x = \xi(t)$ is weaker than that of $|x|^{-1}$. Moreover, we want to consider this problem in higher dimensions. Karch-Zheng estimate their solutions with the singular solutions in the 3D stationary Navier-

Stokes equations (the Landau solutions), so we need other method in higher dimensions.

The second goal is to construct the singular solutions with higher dimensional time-dependent singular sets and to prove the removability of time-dependent singular sets. The way to apply the existence theorem of the solution to the Navier-Stokes equations in the Besov space is simple, but we don't know the asymptotic behavior of the solution at the singular sets. Therefore, to obtain the precise estimates we need to pay attention to the structure of the solution in the same manner with Karch-Zheng. In the heat equation, time-dependent singular sets such as m dimensional submanifold were discussed by Htoo-Takahashi-Yanagida and Takahashi-Yamamoto, which seems to be a clue to solve the problem.