

Research Report (September, 2019 - March, 2020)

In the SGU course of Mathematical Physical Science: April 2017-March 2020

Conferring university	Degree name (by completing a course / by thesis only)	Date of conferment
Waseda University	Doctor of Science (course)	March 15, 2020

Enrollment from
September 2017

Department of Mathematics Hiroyuki TSURUMI

I. List of Papers

1. H. Tsurumi, "The stationary Navier-Stokes equations in the scaling invariant Triebel-Lizorkin spaces", *Differ. Integral Equations* Vol.32 (2019), 323-336.
2. H. Tsurumi, "Ill-posedness of the stationary Navier-Stokes equations in Besov spaces", *J. Math. Anal. Appl.* Vol.475 (2019), 1732-1743.
3. H. Tsurumi, "Well-posedness and ill-posedness of the stationary Navier-Stokes equations in toroidal Besov spaces", *Nonlinearity* Vol.32 (2019), 3798-3819.
4. H. Tsurumi, "Well-posedness and ill-posedness problems of the stationary Navier-Stokes equations in scaling invariant Besov spaces" *Arch. Ration. Mech. Anal.* Vol. 234 (2019), 911-923.
5. H. Tsurumi, "Counter-examples of the bilinear estimates of the Holder type inequality in homogeneous Besov spaces", *Tokyo J. Math.* (2020), in press.
6. H. Tsurumi, "The two-dimensional stationary Navier-Stokes equations in toroidal Besov spaces", submitted.

II. Record of Awards

MSJ Takebe Katahiro Prize for Encouragement of Young Researchers, 2019.

III. List of Talks

1. H. Tsurumi, "Ill-posedness of the stationary Navier-Stokes equations in Besov spaces", IRTG seminar, TU Darmstadt, Germany, October, 2017.
2. H. Tsurumi, "Ill-posedness of the stationary Navier-Stokes equations in homogeneous Besov spaces", 『若手による流体力学の基礎方程式研究集会』, Nagoya University, January, 2018.
3. H. Tsurumi, "Well-posedness and ill-posedness of the stationary Navier-Stokes equations in Triebel-Lizorkin spaces", The 15th Japanese-German International Workshop on Mathematical Fluid Dynamics, Waseda University, January, 2018.
4. H. Tsurumi, "Solutions of the stationary Navier-Stokes equations in homogeneous Besov and Triebel-Lizorkin spaces", 『関数空間の深化とその周辺』, Kyoto University, February, 2018.
5. H. Tsurumi, "Solutions of the stationary Navier-Stokes equations in homogeneous Besov and Triebel-Lizorkin spaces", 『若手のための偏微分方程式と数学解析』, Fukuoka University, February, 2018.
6. H. Tsurumi, "Well-posedness and ill-posedness of the stationary Navier-Stokes equations in Besov spaces", Japanese-Indonesian International Workshop on Mathematical Fluid Dynamics, Waseda University, March, 2018.
7. H. Tsurumi, "Solutions of the stationary Navier-Stokes equations in homogeneous Triebel-Lizorkin spaces" and "Ill-posedness of the stationary Navier-Stokes equations in homogeneous Besov spaces", MSJ Spring Meeting, Tokyo University, March, 2018.
8. H. Tsurumi, "Solutions of the stationary Navier-Stokes equations in homogeneous Besov and Triebel-Lizorkin spaces", 名古屋微分方程式セミナー, Nagoya University, May, 2018.

9. H. Tsurumi, "Counter examples of the bilinear estimates of the H¹-older type inequality in homogeneous Besov spaces" and "Ill-posedness of the stationary Navier-Stokes equations in scaling invariant homogeneous Besov spaces", MSJ Autumn Meeting, Okayama University, September, 2018.
10. H. Tsurumi, "Well-posedness and ill-posedness of the stationary Navier-Stokes equations in scaling invariant Besov spaces", International Conferences on PDEs from fluids, Wuhan University, China, October, 2018.
11. H. Tsurumi, "Well-posedness and ill-posedness problems of the stationary Navier-Stokes equations in scaling invariant Besov spaces", RIMS 『関数空間の一般化とその周辺』, Kyoto University, November, 2018.
12. H. Tsurumi, "On the ill-posedness of the stationary Navier-Stokes equations in scaling invariant Besov spaces", RIMS 『Mathematical Analysis of Viscous Incompressible Fluid』, Kyoto University, December, 2018.
13. H. Tsurumi, "Well-posedness and ill-posedness of the stationary Navier-Stokes equations in the scaling invariant Besov space", 信州微分方程式セミナー, Shinshu University, December, 2018.
14. H. Tsurumi, "Besov 空間における定常 Navier-Stokes 方程式の適切・非適切性", 第 41 回発展方程式若手セミナー, Gunma Pref., August, 2019.
15. H. Tsurumi, "On the ill-posedness of the stationary Navier-Stokes equations in Besov spaces", 微分方程式の総合的研究, Tokyo Institute of Technology, December, 2019.

IV. Research Results in AY2019

I considered the stationary Navier-Stokes equations in homogeneous Besov spaces on two dimensional tori. For any $\varepsilon > 0$ and $(p, q) \in ([1, 2) \times [1, \infty]) \cup (\{2\} \times [1, 2])$, we showed the existence, uniqueness, and continuous dependency of solutions in $\dot{B}_{p+\varepsilon, q}^{-1+2/p}$ for given small external forces in $\dot{B}_{p+\varepsilon, q}^{-3+2/p}$. These spaces become closer to the scaling invariant ones if the difference ε becomes smaller. Furthermore, we showed that such well-posedness property is not generally guaranteed in the rest case of (p, q) , by constructing counter examples.

I am now considering the above equation in homogeneous Besov spaces on the whole 2D plane with Dr. Xin Zhang (researcher at Waseda Research Institute for Science and Engineering).

V. Summary (From April 2017 to May 2020)

In my doctoral course, I studied the well and ill-posedness of the stationary Navier-Stokes equations. Based on the previous study of Mr. Kenta Kaneko, a senior in the laboratory, I obtained a clear borderline between the well-posedness and ill-posedness in the framework of Besov space.

From October to December in 2017, I spent a short time studying at TU Darmstadt in Germany. In there, Dr. Patrick Tolksdorf and Dr. Martin Saal, who were postdoctoral fellows at there, kindly gave me a good suggestion for my study on Triebel-Lizorkin spaces and toroidal Besov spaces. For this study abroad, I would like to thank Ms. Natsuko Ikezaki, who was a secretary in this course, for arranging air tickets and office procedures.

In this course, through several seminars and lectures, I interacted with professors and students who are studying in various fields. In particular, through the lectures by Prof. Guest Martin, Prof. Hiroaki Yoshimura, and Dr. Linyu Peng, I learned the basics and latest research trends of dynamical systems and geometry. It was a very good opportunity for me to touch on other fields and reconsider the significance of my major.

In addition, I received opportunities of presentation at international research conferences. My presentation there led to the joint work with Dr. Xin Zhang mentioned above. Through this course, we have had many valuable experiences that will lead to my future research activities. I am deeply grateful to professors involved in this course and Ms. Yukari Ishizaki (related secretary), and I would like to get involved in this course even after graduation.