

Research Report (April, 2020 - March, 2021)

In the SGU course of Mathematical Physical Science: April 2018-March 2021

Conferring university	Degree name (by completing a course / by thesis only)	Date of conferment
Waseda University	Doctor of Science (course)	March 15, 2021

Enrollment from
April 2018

Department of Pure and Applied Mathematics

Ryo Kanamaru

I. List of Papers

1. Farwig, R., Kanamaru, R., Optimality of Serrin type extension criteria to the Navier-Stokes equations, *Advances in Nonlinear Analysis*, Published online 5 March 2021, Volume 10, Issue 1, 1071-1085.
DOI: <https://doi.org/10.1515/anona-2020-0130>
2. Kanamaru, R., Optimality of logarithmic interpolation inequalities and extension criteria to the Navier-Stokes and Euler equations in Vishik spaces, *Journal of Evolution Equations*, published online 1 February 2020, Volume 20, Issue 4, 1381-1397, 1 December 2020.
DOI: <https://doi.org/10.1007/s00028-020-00559-0>
3. Kanamaru, R., Brezis-Gallouet-Wainger type inequalities and a priori estimates of strong solutions to Navier-Stokes equations, *Journal of Functional Analysis*, published online 26 July 2019, Volume 278, Issue 4, 1 March 2020.
DOI: <https://doi.org/10.1016/j.jfa.2019.108277>

4. Record of Awards

Non.

5. List of Talks

1. Optimality of extension and regularity criteria on the Navier-Stokes equations, *International Workshop on Multiphase Flows: Analysis, Modelling and Numerics*, Waseda University, December 1st-4th, 2020.
2. Optimality of logarithmic interpolation inequalities and extension criteria to the Navier-Stokes and Euler equations in Vishik spaces, *Oberseminar Analysis*, Technische Universität Darmstadt, December 18th, 2019.
3. Extension criteria of strong solutions to the Navier-Stokes and Euler equations, 第41回発展方程式若手セミナー, Gunma Ikaho 温泉旅館ふくぜん, August 26th-29th, 2019.
4. Improvement of the extension theorem of strong solutions to Navier-Stokes equations by Vishik type spaces, *MSJ Spring Meeting 2019*, Tokyo Institute of Technology, March 17th-20th, 2019.
5. Improvement of the extension theorem of strong solutions to Navier-Stokes equations by Vishik type spaces, 研究集会「若手のための偏微分方程式と数学解析」, Fukuoka University,

February 13th-14th, 2019.

6. Brezis-Gallouet-Wainger type inequalities and a priori estimates of time local strong solutions to Navier-Stokes equations, MSJ Autumn Meeting 2018, Okayama University, September 24th-27th, 2018.

6. Research Results in AY2020

We showed logarithmic interpolation inequalities by means of a Besov-Vishik space $B_{p,q,\beta}^s$ and a Triebel-Lizorkin-Vishik space $F_{p,q,\beta}^s$ which are larger than the homogeneous Besov space $\dot{B}_{p,q}^s$ and the homogeneous Triebel-Lizorkin space $\dot{F}_{p,q}^s$, respectively. We see that these spaces are the weakest normed spaces that satisfy the logarithmic interpolation inequalities. As an application of this inequality, we consider the problems on extension of strong solutions and regularity of weak solutions to the Navier-Stokes equations for the viscous incompressible fluid and the Euler equations for the ideal incompressible fluid. The aim of this study is to improve the Beale-Kato-Majda, Beirão da Veiga and Serrin type criteria by means of $B_{p,q,\beta}^s$ and $F_{p,q,\beta}^s$. Furthermore, we established a new a priori estimate of strong solutions to the Navier-Stokes equations which has an almost single exponential growth form with respect to the scaling invariant quantity of the vorticity.

7. Summary (From April 2018 to May 2021)

In my doctoral course, I studied the non-stationary Navier-Stokes equations for the viscous incompressible fluid in an n -dimensional domain Ω . In particular, I considered the problems on extension and regularity of solutions which are related to the Beale-Kato-Majda, Beirão da Veiga and Serrin type criteria.

From December in 2019 to March in 2020, I spent a short time studying at Technische Universität Darmstadt in Germany. I am deeply grateful to Prof. Reinhard Farwig (Technische Universität Darmstadt), who is my host professor of my study abroad, for giving me such a great experience in Darmstadt. As for the study on Serrin-type criterion, he kindly reviewed my paper and encouraged me in my study. For this study abroad, I would like to thank Ms. Yukari Ishizaki, who is a secretary of Prof. Shibata, for arranging air tickets and office procedures.

In this course, through several seminars and lectures, I interacted with professors and students who are studying in various fields. In particular, I learned the latest research trends of non-linear partial differential equations through the lectures by Prof. Shuichi Kawashima, Prof. Mads Kyed, Prof. Hideo Kozono and Prof. Yoshihiro Shibata. It was a very good opportunity for me to touch on other research methods and develop the study of my major.

In addition, an international conference concerning the unique existence of strong solutions and its asymptotic behaviors of the equations describing the motion of the fluids was held as a webinar via Zoom in December 2020. I received an opportunity of presentation at this conference and had a vigorous discussion with other researchers.

Finally, I would also like to express my gratitude to professors involved in this course and Ms. Yukari Ishizaki.