Research Report (April, 2019 - March, 2020)

Enrollment from Department of Pure and Applied Mathematics Ryo Kanamaru

I. List of Papers

- 1. Farwig, R., Kanamaru, R.: Optimality of Serrin type extension criteria to the Navier-Stokes equations. Indiana Univ. Math. J. (submitted)
- Kanamaru, R.: Brezis-Gallouet-Wainger type inequalities and a prior estimates of strong solutions to Navier-Stokes equations. J. Funct. Anal. **278** (2020) https://doi.org/10.1016/j.jfa.2019.108277
- 3. Kanamaru, R.: Improvement of the extension criterion on strong solutions to the Navier-Stokes equations in Vishik type spaces. Indiana Univ. Math. J. (submitted)
- Kanamaru, R.: Optimality of logarithmic interpolation inequalities and extension criteria to the Navier-Stokes and Euler equations in Vishik spaces. J. Evol. Equ. (to apper) https://doi.org/10.1007/s00028-020-00559-0

II. List of Talks

- Brezis-Gallouet-Wainger type inequalities and a prior estimates of time local strong solutions to Navier-Stokes equations, MSJ Autumn Meeting 2018, Okayama University, September 24th-27th, 2018.
- 2. Improvement of the extension theorem of strong solutions to Navier-Stokes equations by Vishik type spaces, 研究集会「若手のための偏微分方程式と数学解析」, Fukuoka University, February 13th-14th, 2019.
- 3. Improvement of the extension theorem of strong solutions to Navier-Stokes equations by Vishik type spaces, MSJ Spring Meeting 2019, Tokyo Institute of Technology, March 17th 20th, 2019.
- 4. Extension criteria of strong solutions to the Navie-Stokes and Euler eqiations, 第41回発展方 程式若手セミナー, Gunma Ikaho 温泉旅館ふくぜん, August 26th-29th, 2019.
- 5. Optimality of logarithmic interpolation inequalities and extension criteria to the Navier-Stokes and Euler equations in Vishik spaces, Oberseminar Analysis, Technical University Darmstadt, December 18th, 2019.

II. Research Results in AY2019

We proved the logarithmic interpolation inequalities by means of function spaces $\dot{V}_{p,q,\theta}^{s}$, $\dot{U}_{p,\beta,\sigma}^{s}$ which are in some cases larger than $\dot{B}_{p,q}^{s}$. We see that these spaces are optimal ones that satisfy such inequalities, respectively. As an application of those inequalities, we showed that strong solutions to the Navier-Stokes and Euler equations can be extended if a scaling invariant quantity is finite. Roughly speaking, those new criteria may be regarded as optimal ones that guarantee a priori estimates of strong solutions with double exponential growth form.

IV. Research Plan for AY2020

We will consider regularity criteria on strong and weak solutions by applying logarithmic interpolation inequalities to other equations, such as the MHD, Boussinesq, quasi-geostrophic, Cahn-Hilliard and harmonic-heat-flow equations.