中里　研究活動　2017 年度報告

Publications

[1] Lindbladian purification
Christian Arenz, Daniel Burgarth, Vittorio Giovannetti, Hiromichi Nakazato and Kazuya Yuasa
Quantum Sci. Technol. **2** (2017, April) 024001 (10 pages)

[2] Synchronizing Quantum Harmonic Oscillators through a Two-Level Systems
B. Militello, H. Nakazato and A. Napoli
Phys. Rev. A **96** No. 2 (2017, August) 023862 (8 pages)

[3] An example of interplay between Physics and Mathematics: Exact resolution of a new class of Riccati Equations
L. A. Markovich, R. Grimaudo, H. Nakazao and A. Messina
Ann. Phys. **385** (2017, October) 522–531

[4] Completeness of scattering states of the Dirac Hamiltonian with a step potential
M. Ochiai and H. Nakazato
J. Phys. Commun. **2** No. 1 (2018, January) 015006 (16 pages)

Research summary in 2017

1 The idea of 'Hamiltonian purification' is known as : any Hamiltonians of a quantum system, which are in general noncommutative with each other, can be made commutative by embedding them in an extended Hilbert space, thus the dynamics in the extended space becomes trivial and simple. The original noncommutative Hamiltonians are recovered by projecting the system back onto the original Hilbert space through frequent measurements. Here, we generalize this idea to open system dynamics by presenting a simple construction to make Lindbladians, as well as Hamiltonians, commutative on a larger space with an auxiliary system. We show that the original dynamics can be recovered through frequently measuring the auxiliary system in a non-selective way.
「ハミルトニアン純粋化」とは，互いに非可換な複数のハミルトニアンによって制御される量子系のダイナミクスは，ハミルトニアンを拡張されたヒルベルト空間に埋め込めば可換とすることが常に可能であり，単純で自明なものとできることを意味する．このとき非可換なダイナミクスは元のヒルベルト空間への頻繁な射影によって回復される．ここではこの考えを開放量子系に拡張し，補助系を伴ったより大きな空間において可換なリンドブラディアンあるいはハミルトニアンを構成する方法を提案した．元の非可換ダイナミクスは，補助系に非選択的測定をすることで再現される．

2 Two oscillators coupled to a two-level system which in turn is coupled to an infinite number of oscillators (reservoir) are considered, bringing to light the occurrence of synchronization. A detailed analysis clarifies the physical mechanism that forces the system to oscillate at a single frequency with a predictable and tunable phase difference. Finally, the scheme is generalized to the case of N oscillators and $M(< N)$ two-level systems.
熱浴中におかれた量子 2 準位系に結合した 2 つの調和振動子系の量子共鳴現象を考察し，系が予言可能で調整可能な位相差をもった単一振動に導かれる物理過程を明らかにした．また，M 個の 2 準位系と N 個の調和振動子系への一般化も行った．

3 A novel recipe for exactly solving in finite terms a class of special differential Riccati equations is reported. Our procedure is entirely based on a successful resolution strategy quite recently applied to quantum dynamical time-dependent SU(2) problems. The general integral of exemplary differential Riccati equations, not previously considered in the specialized literature, is explicitly determined to illustrate both mathematical usefulness and easiness of applicability of our proposed treatment. The possibility of exploiting the general integral of a given differential Riccati equation to solve an SU(2) quantum dynamical problem, is succinctly pointed out.
特定のクラスのリカッティ方程式を，有限の項をもって正確に解く全く新しい手法を提案した．これは，時間依存する SU(2) 量子ダイナミクスの有効な解法として最近見出された手法に全面的に依拠している

が，従来知られていなかったリカッティ方程式の新たな解を明示し，この手法の数学的有効性と取り扱いやすさを例示するものとなっている．与えられたリカッティ方程式の積分を利用して SU(2) 量子ダイナミクスを解く可能性にも簡潔に触れている．

4 The completeness, together with the orthonormality, of the eigenfunctions of the Dirac Hamiltonian with a step potential is shown explicitly. These eigenfunctions describe the scattering process of a relativistic fermion off the step potential and the resolution of the identity in terms of them (completeness) is shown by explicitly summing them up, where appropriate treatments of the momentum integrations are crucial. The result would bring about a basis on which a field theoretical treatment for such a system can be developed.
階段型ポテンシャルを伴った Dirac ハミルトニアンの固有状態の完全性と正規直交性を，運動量積分を陽に実行することで直接示した．これらの固有関数は相対論的フェルミ粒子が階段型ポテンシャルによって散乱される過程を記述しており，完全正規直交性の証明は，この系に対する場の理論的取り扱いの基礎となるものである．