

## SGU Module: Advanced Study of Nonlinear Mechanics Spring Semester of 2018

### (1) Symmetries and Conservation Laws: Differential and Difference Equations (Instructor: Linyu Peng, Waseda University)

**Abstract:** Many techniques for solving particular ordinary differential equations, e.g. separable or homogeneous equations, are special cases of a general integration method based on the invariance of differential equations under a continuous group of symmetries. These are called Lie groups of symmetries, or Lie symmetries for short. In this part of courses, applications of Lie groups of symmetries will be our main interests; in particular, we will study integration and reduction methods, group-invariant (or similarity) solutions of partial differential equations, conservation laws and Noether's theorems, integrable equations which can be reduced to Painlevé equations or which admit infinitely many symmetries or conservation laws. At the end, their difference counterparts will be mentioned if time permits. During the lectures, some open questions will be addressed.

#### Contents:

1. Calculation of symmetries for differential equations
2. Integration and reduction of differential equations
3. Group-invariant/similarity solutions of partial differential equations
4. Variational symmetries and conservation laws
5. Integrable systems: Infinitely many symmetries or conservation laws
6. Symmetry methods for difference equations

#### Schedule:

May 28th	Mon. (月)	1445 -- 1800	(4-5 限)	51-805
30th	Wed. (水)	1445 -- 1800	(4-5 限)	51-711
June 1st	Fri. (金)	1445 -- 1800	(4-5 限)	51-805
4th	Mon. (月)	1445 -- 1800	(4-5 限)	51-805
6th	Wed. (水)	1445 -- 1615	(4 限)	51-711

## (2) Dirac geometry, variational principles, and dynamical systems

(Instructor: Hiroaki Yoshimura, Waseda University)

**Abstract:** It is well known that symplectic geometry provides an underlying geometric structure of Hamiltonian and Lagrangian systems, in which there exist the variational principles associated with these dynamical formulations. On the other hand, these investigations in the context of symplectic geometry are mainly restricted to the canonical cases of unconstrained systems with non-degenerate symplectic structures. Recently, it has been clarified that the notion of Dirac structures, which is a geometric object of unifying pre-symplectic and almost Poisson structures, plays an essential role in treating more general cases of nonholonomic and degenerate Lagrangian systems. In this short course, we make a short introduction to Dirac geometry and its applications to nonholonomic mechanics. First, we make brief reviews on the geometric approaches to the canonical Hamiltonian and Lagrangian systems, together with the variational formulations. Then, we introduce the notion of Dirac structures and with the associated dynamical systems, into which nonholonomic distributions are easily incorporated. Furthermore, we show the variational link with the Dirac structures and the associated Dirac dynamical systems. Lastly, we will illustrate the theory of Dirac geometry and dynamical systems by some examples of nonholonomic systems and electric circuits.

### Contents:

1. Some reviews on Hamiltonian and Lagrangian mechanics
2. Dirac geometry and the associated dynamical systems
3. Variational structures of Dirac dynamical systems
4. Some applications to nonholonomic mechanics

### Schedule:

June 8th	Fri. (金)	1445 -- 1800	(4-5 限)	51-805
22nd	Fri. (金)	1445 -- 1800	(4-5 限)	51-805
29th	Fri. (金)	1445 -- 1800	(4-5 限)	51-805