Mathematics and Physics Unit "Multiscale Analysis, Modeling and Simulation"
Top Global University Project, Waseda University

International Workshop on "Fundamental Problems in Mathematical and Theoretical Physics"

Date: July 24 – July 28, 2017
Venue: Conference Room No. 2, 1st Floor, 55N Bldg., Nishi-Waseda Campus, Waseda University
早稲田大学 西早稲田キャンパス **55** 号館 **N** 棟 **1** 階 第 **2** 会議室

Part I. Quantum Physics

◆Saverio PASCAZIO (University of Bari, Italy)◆

◆Mini Course Ⅰ	July 24, Monday	10:30 – 12:30
◆Mini Course Ⅱ	July 24, Monday	16:30 – 18:00
◆Mini Course Ⅲ	July 25, Tuesday	14:45 – 16:15

Wave Propagation: From d'Alembert to Dirac

The mechanism of wave propagation has fascinated mathematicians and physicists for centuries. Waves characterize many physical phenomena and arise in many fields: familiar examples are sound in acoustics, light in electromagnetism, and ocean waves in fluid dynamics. In quantum mechanics, waves coexist with particles (giving rise to the so-called wave-particle dualism).

The one-dimensional wave equation was first understood and discovered by Jean le Rond d'Alembert in 1746. A few years later, Leonhard Euler wrote the three-dimensional wave equation. The main mathematical and physical ingredients at the origin of wave propagation were also studied by Daniel Bernoulli and Joseph-Louis Lagrange. They were all trying to understand vibrating strings and sound propagation in musical instruments.

James Clerk Maxwell first predicted the existence of electromagnetic waves, by careful examination of the equations of electromagnetism. The unification of light and electrical phenomena was one of the greatest achievements of the 19th century.

Mathematically, the wave equation is a second-order linear hyperbolic partial differential equation. As such, it models processes which evolve over time. It can be rigorously studied, and admits existence and unicity theorems. However, when a wave meets an obstacle, part of it is reflected off the surface of the material, while some is transmitted through the material, and the problem often becomes too complicated to admit analytic (simple) solutions. In such a case, a famous principle is often invoked, that enables one to guess the form of the solution.

Every point on the wave front of a propagating wave is a source of secondary wavelets, which spread forward at the same speed as the source wave. The wave front at later times is then given by the surface tangent to the secondary wavelets. This principle was proposed by Christiaan Huygens in 1678, to explain the laws of reflection and refraction. It was used again more than a century later, in 1816, by Augustin-Jean Fresnel, to interpret the diffraction effects that occur when visible light encounters slits, edges and screens.

The principle provides crucial insight into the nature of wave propagation and it is a milestone in the physics of undulatory phenomena. For this reason, its universal validity is usually taken for granted. However, yet one century later, Jacques Hadamard noticed that Huygens' principle is valid only when waves propagate in an odd number $n > 1$ of spatial dimensions. The principle is therefore neither valid for $n = 1$, nor for even n.

Both quantum mechanics and quantum field theory make use of wave equations in their formulation. It is therefore interesting to ask whether Huygens' principle holds for the seminal equations that are the backbone of these theories. The Schrödinger equation, being non-relativistic, does not admit a satisfactory formulation of this question. What about the Dirac equation?

We shall discuss the validity of Huygens' principle for the massless Dirac-Weyl equation. We shall find that the principle holds for odd space dimension n, while it is invalid for even n. We explicitly discuss the cases $n = 1,\ 2$ and 3.

◆Paolo FACCHI (University of Bari, Italy)◆

◆Mini Course Ⅰ	July 24, Monday	14:45 – 16:15
◆Mini Course Ⅱ	July 25, Tuesday	10:30 – 12:30

Operator Semigroups and Product Formulae

One-parameter semigroups of transformations describe the time evolution of autonomous deterministic systems. The concept of semigroups encompasses the structure of the solutions of initial-value problems for ordinary and partial differential equations. It ranges from the heat and the wave equation of classical physics, up to the Schrödinger and the master equation of quantum physics. The continuity property of these semigroups is the key to a deep and beautiful theory, that has become an indispensable tool in a great number of areas of modern mathematics and physics.

In these lectures I will first introduce the basic properties of strongly continuous semigroups of linear operators on Banach spaces. Then I will touch upon perturbation theory and Dyson expansion. Finally we will have a look at approximation theory and at limit product formulae, ranging from Trotter-Kato to the quantum Zeno effect.

◆Daniel BURGARTH (Aberystwyth University, UK)◆

◆Mini Course Ⅰ	July 25, Tuesday	16:30 – 18:00
◆Mini Course Ⅱ	July 26, Wednesday	10:30 – 12:00

Introduction to Completely Positive Maps

The natural generalization of a probability distribution to quantum mechanics is the density matrix, which is a positive matrix with unit trace. Stochastic maps are generalized as completely positive maps, which have a rich and beautiful structure, and map density matrices to density matrices. The purpose of these lectures is to introduce some of the interesting properties such maps have and outline some applications in quantum theory.