量子物理学特別講義 Special Lecture on Quantum Physics

GROUND STATES and LONG TIME DYNAMICS FOR FRACTIONAL TYPE NLS and HALF WAVE EQUATION

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① July 20 (THU) 10:40-12:10	② July 21 (FRI) 13:00-14:30
③ July 21 (FRI) 14:45-16:15	④ July 22 (SAT) 13:00-14:30

1 Ground states for the fractional Hartree Model

In the first part of the lecture course we consider the fractional Hartree model, with general power non-linearity and space dimension. We construct variationally the "normalized" solutions for the corresponding Choquard-Pekar model - in particular a number of key properties, like smoothness and bell-shapedness are established. As a consequence of the construction, we show that these solitons are spectrally stable as solutions to the time-dependent Hartree model.

In addition, we analyze the spectral stability of the Moroz-Van Schaftingen solitons of the classical Hartree problem, in any dimensions and power non-linearity. A full classification is obtained, the main conclusion of which is that only and exactly the "normalized" solutions (which exist only in a portion of the range) are spectrally stable.

2 Local uniqueness of groung states for the generalized p-Hartree model

In the second part of the lecture course we consider the generalized p-Hartree-Choquard equation in 3 dimensional case and the corresponding Weinstein type functional. The study of orbital stability of the corresponding minimizers depends essentially in the local uniqueness of these minimizers.

In equivalent way one can minimize the energy functional subject to the constraint fixing the L^2 norm. The uniqueness of the minimizers for the case $p = 2$, i.e. for the case of the Hartree-Choquard is well known. The main difficulty for the case $2 < p < 7/3$ is connected with the control of the L^p norm of the minimizers.

Our approach is based on the Weinstein method and appropriate study of the spectral properties of the operator L_+ crucial in the Weinstein approach.

3 Stability/instability for fractional Schrödinger equations

We consider the Cauchy problems associated with semirelativistc NLS (sNLS) and half wave (HW). In particular we focus on the following two main questions: local/global Cauchy theory; existence and stability/instability of ground states. In between other results, we prove the existence and stability of ground states for sNLS in the L^2 supercritical regime. This is in sharp contrast with the instability of ground states for the corresponding HW, which will be established too, by showing an inflation of norms phenomenon. Concerning the Cauchy theory we show, under radial symmetry assumption the following results: a local existence result in H^1 for energy subcritical nonlinearity and a global existence result in the L^2 subcritical regime.

4 Blow - up phenomena for the fractional Landau - Ginzburg equation

In the last part of the lecture course we consider a variant of NLS that is known as kind of Landau - Ginzburg equation that is closely connected with the Kuramoto system, intensively studied in connection with some biomathematical models.
The fractional dynamics seems more adapted to synchronization models, therefore we can consider the fractional Ginzburg - Landau equation with repulsive self interacting term. We shall discuss a blow up result assuming initial data are in $H^s(\mathbf{R}^n)$ and $s > n/2$.