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スーパーグローバル大学創成支援 早稲田大学 数物系科学拠点
Mathematics and Physics Unit "Multiscale Analysis, Modelling and Simulation"
Top Global University Project, Waseda University

International Workshop on Multiphase Flows: Analysis, Modelling and Numerics

Waseda University, Tokyo, Japan 1-4 December, 2020

Venue: Online via Zoom

Program (in Japan Time = GMT+9)

	Tue., 1 Dec.		Wed., 2 Dec.	Thu., 3 Dec.		Fri., 4 Dec.
13:00 13:50	**Masahiro SUZUKI**	13:00 13:50	**Ryosuke NAKASATO**	**Tatsu-Hiko MIURA**	9:00 9:50	**Giovanni P. GALDI**
14:00 14:50	**Ken ABE**	14:00 14:50	**Keiichi WATANABE**	**Mitsuo HIGAKI**	10:00 10:50	**Kazuyuki TSUDA**
15:00 15:50	**Takayuki KUBO**	15:00 15:50	**Yasunori Maekawa**	**Tsuyoshi YONEDA**	11:00 11:50	**Hirokazu SAITO**
	Student Session				12:00 12:50	**Tsukasa IWABUCHI**
16:00 16:20	**Kosuke KITA**	16:00 16:20	**Jumpei INOUE**	**Takahiro USHIOKU**		
16:30 16:50	**Ryo KANAMARU**	16:30 16:50	**Fumitaka WAKABAYASHI**	**Masahito WATANABE**		
17:00 17:20	**Yoshiki KANEKO**					
	Oxford-Waseda in Mathematics					
18:20 19:00	Opening	18:00 18:50	**Endre SÜLI**	**Hiroaki YOSHIMURA**		
19:10 20:00	**Gregory SEREGIN**	19:00 19:50	**Tadahisa FUNAKI**	**Jan KRISTENSEN**		
20:10 21:00	**Tohru OZAWA**	20:00 20:50	**Gui-Qiang G. CHEN**	**Hideo KOZONO**		

Abstracts

Tuesday, 1st December 2020

Derivation of the Ion Equation

Masahiro Suzuki

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The classical Euler-Poisson system for electrons and ions, interacting through an electrostatic field, describes the motion of plasma. In a certain situation, physicists use a simpler system only for ions by assuming the Boltzmann relation. The simpler system can be derived by letting the mass ratio of an electron and an ion tend to zero. We justify this limit rigorously.

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Existence of Vortex Rings in Beltrami Flows

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Traveling waves of the Euler equations are related with translating vortex motion dating back to a pioneering work of Helmholtz in 1858. Because of the 3d nature of the domain, a vortex can be supported on a knotted solid torus and translate by a constant speed. Existence of such a vortex is a conjecture of Kelvin. I will explain existence of traveling wave solutions to the Euler equations forming axisymmetric Beltrami fields with a non-constant proportionality factor. They form a vortex ring with nested invariant tori consisting level sets of the proportionality factor.

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Analysis of Non-Stationary Navier-Stokes Equations Approximated by the Pressure Stabilization Method

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Let us consider the non-stationary Navier-Stokes equations approximated by the pressure stabilization method. As you know, One of the difficulty of analysis for the Navier-Stokes equations is the incompressible condition $\nabla \cdot u = 0$. In order to overcome this difficulty, we often use Helmholtz decomposition. On the other hand, in numerical analysis, some penalty methods are employed. In this talk, we consider the Navier-Stokes equations with incompressible condition approximated by pressure stabilization method, which is one of the penalty method. Namely, we consider the following approximated incompressible condition instead of usual incompressible condition:

$$(u, \nabla\varphi) = \alpha^{-1}(\nabla\pi, \nabla\varphi) \qquad (\forall\varphi \in \widehat{W^1_{q'}}(\Omega)),$$

where α is a perturbation parameter. As $\alpha \to \infty$, the approximated incompressible condition tends to usual incompressible condition formally.

In this talk, we report the local in time existence theorem for solutions to our problem and the error estimate in the L_p in time and the L_q in space framework with $n/2 < q < \infty$ and $\max(1, n/q) < p < \infty$. Moreover we shall report on recent results. This talk are based on some results obtained in our joint work with R. Matsui and H. Kikuchi.

References

[1] T. Kubo and R. Matsui, On pressure stabilization method for nonstationary Navier-Stokes equations, *Communications on Pure and Applied Analysis*, **17**, No.6 (2018), 2283-2307.

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Comparison Theorem for Parabolic Equations Governed by Nonlinear Boundary Conditions and Its Applications

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We are concerned with the comparison theorem of the initial-boundary value problem for nonlinear parabolic equations governed by the nonlinear boundary conditions of radiation type. It is well known that classical comparison principle is a useful tool in the study of reaction diffusion equations. This classical result claims that if two initial data satisfy some order then the corresponding solutions keep the initial data order. In this talk, we clarify the relationship of two solutions of reaction diffusion equations governed by the different nonlinear boundary conditions. This talk is based on a joint work with Prof. Mitsuharu Ôtani.

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Optimality of Extension and Regularity Criteria on the Navier-Stokes Equations

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In this talk, we consider the extension criterion of strong solutions and the regularity criterion of weak solutions for the Navier-Stokes equations. To obtain our extension principle, we need logarithmic interpolation inequalities by means of the Besov-Vishik space $\mathcal{B}^s_{p,q,\beta}$ and the Triebel-Lizorkin-Vishik space $\mathcal{F}^s_{p,q,\beta}$ which are larger than the homogeneous Besov space $\dot{B}^s_{p,q}$ and the homogeneous Triebel-Lizorkin space $\dot{F}^s_{p,q}$, respectively. We see that $\mathcal{B}^s_{p,q,\beta}$ and $\mathcal{F}^s_{p,q,\beta}$ are the optimal spaces that satisfy the logarithmic interpolation inequalities. As a byproduct of our extension theorem, we prove that a weak solution is smooth if a scaling invariant quantity of the solution is finite. Namely, the Beale-Kato-Majda, Beirão da Veiga and Serrin type regularity criteria are improved in terms of $\mathcal{B}^s_{p,q,\beta}$ and $\mathcal{F}^s_{p,q,\beta}$.

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Solutions of the tt*-Toda Equations and Quantum Cohomology of Flag Manifolds

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We introduce the tt*-Toda equations for general complex simple Lie groups. The tt*-Toda equations are special cases of the topological-antitopological fusion equations which are introduced by Cecotti and Vafa. It is known that we obtain a solution of the tt*-Toda equation from the quantum cohomology of the projective spaces when the Lie group is $SL(n+1, C)$. In this talk, we see that we obtain local solutions to the tt*-Toda equations from the quantum cohomology of minuscule flag manifolds.

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Local Regularity of Axisymmetric Solutions to the Navier-Stokes Equations

Gregory Seregin

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In the talk, a local regularity condition for axisymmetric solutions to the non-stationary 3D Navier-Stokes equations is presented. It reads that axially symmetric energy solutions to the Navier-Stokes equations have no Type I blowups.

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On the Poincaré and Related Inequalities

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Sharp remainder formulae are presented for the Poincaré and related inequalities. This talk is based on a recent joint work with Durvudkhan Suragan.

References

[1] T. Ozawa and D. Suragan, Sharp remainder of the Poincaré inequality, *Proc. AMS*, **148**, (2020), 4235-4239.

[2] T. Ozawa and D. Suragan, Poincaré inequalities with exact missing terms on homogeneous groups, *J. MSJ*, (in press).

[3] T. Ozawa and D. Suragan, Representation formulae for the higher-order Steklov and L^{2^m}-Friedrichs inequalities, preprint.

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Wednesday, 2nd December 2020

Well-Posedness and Time-Decay Estimates for the Compressible Hall-Magnetohydrodynamic System in the Critical L^2 Framework

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We consider the following initial value problem for the compressible Hall-magnetohydrodynamic system, which is a hyperbolic-parabolic system of conservation laws with non-symmetric diffusion:

$$\begin{cases} \partial_t \widetilde{\rho} + \operatorname{div}(\widetilde{\rho} u) = 0, & t>0,\ x\in\mathbb{R}^3, \\ \partial_t(\widetilde{\rho} u) + \operatorname{div}(\widetilde{\rho} u\otimes u) + \nabla P(\widetilde{\rho}) = \mathcal{L}u + (\nabla\times\widetilde{B})\times\widetilde{B}, & t>0,\ x\in\mathbb{R}^3, \\ \partial_t\widetilde{B} + \sigma_H \nabla\times\left(\dfrac{(\nabla\times\widetilde{B})\times\widetilde{B}}{\widetilde{\rho}}\right) = \sigma_D\Delta\widetilde{B} + \nabla\times(u\times\widetilde{B}), & t>0,\ x\in\mathbb{R}^3, \\ \operatorname{div}\widetilde{B} = 0, & t>0,\ x\in\mathbb{R}^3, \\ (\widetilde{\rho}, u, \widetilde{B})|_{t=0} = (\widetilde{\rho}_0, u_0, \widetilde{B}_0), & x\in\mathbb{R}^3, \end{cases} \tag{1}$$

where $\widetilde{\rho} = \widetilde{\rho}(t,x) : \mathbb{R}_+\times\mathbb{R}^3 \to \mathbb{R}_+$, $u = u(t,x) : \mathbb{R}_+\times\mathbb{R}^3\to\mathbb{R}^3$ and $\widetilde{B} = \widetilde{B}(t,x) : \mathbb{R}_+\times\mathbb{R}^3\to\mathbb{R}^3$ denote the density of fluid, the velocity of fluid and the magnetic field, respectively. The pressure $P = P(\widetilde{\rho}) : \mathbb{R}_+\to\mathbb{R}$ is assumed to be a smooth function of the density $\widetilde{\rho}$ satisfying $P'(\widetilde{\rho}) > 0$. The elliptic operator $\mathcal{L}u$ is denoted by $\mu\Delta u + (\lambda+\mu)\nabla\operatorname{div} u$ and the Lamé coefficients μ and λ fulfill the standard ellipticity conditions $\mu > 0$ and $\lambda + 2\mu > 0$. The constant $\sigma_H > 0$ denotes the Hall coefficient and $\sigma_D > 0$ is given by $\sigma_D = \frac{1}{\sigma\mu_0}$, where the constants $\sigma > 0$ and $\mu_0 > 0$ denote the electrical conductivity and the magnetic permeability, respectively.

We look for a solutions as a perterbation from a constant equilibrium state $(\bar{\rho}, 0, \bar{B})$, where $\bar{\rho} > 0$ is a constant density, $0 \in \mathbb{R}^3$ is the zero velocity and $\bar{B}\in\mathbb{R}^3$ is a constant magnetic field. In the following, we reformulate the system (1). Introducing $\rho := \widetilde{\rho} - \bar{\rho}$ and $B := \widetilde{B} - \bar{B}$, we see that the system (1) is reformulated as follows:

$$\begin{cases} \partial_t\rho + \bar{\rho}\operatorname{div} u = f & t>0,\ x\in\mathbb{R}^3, \\ \partial_t u - \dfrac{1}{\bar{\rho}}\mathcal{L}u + \dfrac{P'(\bar{\rho})}{\bar{\rho}}\nabla\rho - \dfrac{1}{\bar{\rho}}(\nabla\times B)\times\bar{B} = g, & t>0,\ x\in\mathbb{R}^3, \\ \partial_t B - \sigma_D \text{Ł}B + \dfrac{\sigma_H}{\bar{\rho}}\nabla\times\big((\nabla\times B)\times\bar{B}\big) - \nabla\times(u\times\bar{B}) = \nabla\times h, & t>0,\ x\in\mathbb{R}^3, \\ \operatorname{div} B = 0, & t>0,\ x\in\mathbb{R}^3, \\ (\rho, u, B)|_{t=0} = (\rho_0, u_0, B_0), & x\in\mathbb{R}^3, \end{cases} \tag{2}$$

where f, g, h are the nonlinear terms depending only ρ, u, P' and B.

In this talk, we shall state the results on the well-posedness and time-decay estimates for the solution of (2) in the critical Besov space. This talk is based on the joint work with Profs. Shuichi Kawashima (Waseda University) and Takayoshi Ogawa (Tohoku University).

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On the Moving Contact Line Problem in Cylindrical Domains

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We consider a free boundary problem of the Navier–Stokes equations in the three-dimensional Euclidean space with moving contact line, where the 90°-contact angle condition is posed. We show that for given $T > 0$ the problem is local well-posed on $(0, T)$ provided that the initial data are small. In contrast to the strategy in Wilke (2013), we study the transformed problem in an L^p-in-time and L^q-in-space setting, which yields the optimal regularity of the initial data.

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Prandtl Boundary Layer Expansion in Gevrey Class Around Concave Boundary Layer

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We present a recent progress on the Prandtl boundary layer expansion for the two-dimensional Navier-Stokes equations in the half space under the no-slip boundary condition, which is a classical topic in fluid mechanics and important for the analysis of the flow with a small viscosity. It is known by the pioneering work of Sammartino and Caflisch that the Prandtl expansion is valid in a framework of analytic regularity. Recently it is shown by Gerard-Varet, M., and Masmoudi that the Prandtl expansion is valid in a Gevrey 3/2 class around a time-independent concave shear boundary layer, where the key step is the resolvent analysis of the linearized operator that is reduced to the analysis of the 4th order ODE, called the Orr-Sommerfeld equations. While it has been still open whether Gevrey 3/2 regularity is enough to verify the expansion when the concave shear boundary layer depends on the time variable, or even more generally, when the leading boundary layer is concave but not necessarily a shear type. The main difficulty is that the reduction to the Orr-Sommerfeld equations does not work well in this general case. This talk focuses on recent progress in this direction (joint work with David Gerard-Varet (Paris 7) and Nader Masmoudi (NYU)).

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On the Unboundedness of the Ratio of the Total Population to the Total Resources in the Stationary Logistic Equation

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Around 2010, Wei-Ming Ni proposed the following conjecture concerning the diffusive logistic equation: "The ratio of the total population to the total resources under varying the diffusion and the resources is uniformly bounded above", at the same time, he raised a conjecture that the optimal upper bound of the ratio is equal to 3 in the one-dimensional case. In 2016, Bai-He-Li showed the validity of this conjecture in the one-dimensional case. However, in the higher-dimensional cases, it was shown by I.-Kuto (2020) that this conjecture fails. This talk will show that the ratio is unbounded even in the case when the domain is a higher-dimensional ball.

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Removability of Time-Dependent Singularities in the Stokes Equations

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We handle the solution u of the Stokes equations with time-dependent singularities in a bounded domain Ω with the smooth boundary $\partial\Omega$ in $\mathbb{R}^N$. We say that the the curve $\{\xi(t); 0 < t < T\}$ is a family of removable singularities of u in $\Omega \times (0,T)$ if there exists a classical solution $\bar{u}$ in $\Omega \times (0,T)$ such that $\bar{u} \equiv u$ on $\bigcup_{0<t<T}\{\Omega \setminus \{\xi(t)\}\} \times \{t\}$. It is known that for the heat equation the time-dependent singularity $\xi(t)$ is removable if the singularity of the solution is weaker than the order of the fundamental solution to the Laplace equation, where $\xi(t)$ is locally 1/2- Hölder continuous in $t \in \mathbb{R}$. In this talk, we give a similar condition on the removability of time-dependent singularities $\xi \in C^\alpha([0,T];\Omega)$ $(0 < \alpha \leq 1/2)$ of u at $\{\xi(t)\} \times \{t\}$.

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Navier–Stokes–Fokker–Planck Systems: Existence of Large-Data Global Weak Solutions

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This talk is concerned with the mathematical analysis of a system of nonlinear partial differential equations arising in a class of bead-spring-chain models for dilute polymeric fluids, where long polymer molecules immersed in a viscous compressible or incompressible Newtonian fluid are idealised as linear chains of beads, each with a small mass, which are considered to be points positioned in the flow domain, and are assumed to be connected with elastic springs. Models of this kind involve the coupling of the Navier–Stokes equations with a Fokker–Planck equation, where the Fokker–Planck equation governs the evolution of a probability density function describing the configuration of polymer molecules in the flow domain. We shall review recent results concerning the existence of large-data global weak solutions to these models.

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Quasilinear Stochastic PDE and Its Asymptotics in Noise

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We consider the following quasilinear stochastic PDE (SPDE) on $\mathbb{T} = [0, 1]$ with periodic boundary condition:

$$\partial_t u = a(\nabla u)\Delta u + g(\nabla u)\xi,$$

where $\nabla = \partial_x, \Delta = \partial_x^2$, $a, g \in C_b^3(\mathbb{R})$, $0 < c \leq a(v) \leq C$ and $\xi = \xi(x)$ denotes the noise. We take the spatial white noise as ξ, which satisfies $\xi \in C^\alpha$ for $\alpha < -\frac{1}{2}$ (or $\alpha < -\frac{d}{2}$ in d-dimensional case), where $C^\alpha \equiv C^\alpha(\mathbb{T}) = \mathcal{B}^\alpha_{\infty,\infty}(\mathbb{T})$ stands for the Hölder(-Besov) space with exponent α. In particular, ξ is a genuine generalized function. Roughly by Schauder estimate, we expect $u \in C^{\alpha+2}$ so that $a(\nabla u)$ is well-defined only when $\alpha + 2 \geq 1$, i.e., $d = 1$.

Recently, two different theories for singular SPDEs rapidly develop, the regularity structures due to M. Hairer (2014 Fields medalist) and the paracontrolled calculus due to M. Gubinelli and others. We apply the latter, and show the local-in-time solvability and the convergence of the solutions when ξ moves. The convergence holds when the enhanced noise $\hat{\xi}$, which is a pair of ξ and a certain quadratic function of ξ, converges.

Let φ be an integral of a, that is, $a = \varphi'$ and let $v := \nabla u$. Then, v solves the SPDE

$$\partial_t v = \Delta\big(\varphi(v)\big) + \nabla\big(g(v)\xi\big).$$

In particular, this SPDE with $g = \varphi$ and smeared noise ξ arises naturally in a hydrodynamic scaling limit of a certain interacting particle system in a random environment. In case $g = \varphi$ with spatial white noise ξ, we can show the global-in-time solvability and the convergence of $v(t)$ as $t \to \infty$ to the stationary solution which is unique for each conserved quantity $\int_{\mathbb{T}} v dx$ specified by $v(0)$.

The first part is joint work (arXiv:2005.03326) with M. Hoshino (Kyushu), S. Sethuraman (Arizona), B. Xie (Shinshu) and the second is with B. Xie.

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Multidimensional Transonic Shock Waves, Free Boundary Problems and Nonlinear PDEs of Mixed Type

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In this talk, we will discuss some of the most recent developments in the analysis of multidimensional transonic shock waves and related free boundary problems through several longstanding fundamental shock problems involving both supersonic and subsonic phases in fluid mechanics. The mathematical analysis of these free boundary problems involves dealing with several core difficulties we have to face in the analysis of nonlinear partial differential equations (PDEs). These include nonlinear PDEs of mixed hyperbolic-elliptic type, nonlinear degenerate elliptic PDEs, nonlinear degenerate hyperbolic PDEs, corner singularities (especially when free boundaries meet the fixed boundaries where the nonlinear PDEs experience their degeneracy), among others. These difficulties also arise in many further fundamental problems in continuum mechanics, mathematical physics, differential geometry, and other areas. Some further developments, open problems, and mathematical challenges in this direction are also addressed.

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Thursday, 3rd December 2020

Rigorous Derivation of the Surface Navier-Stokes Equations by the Thin-Film Limit

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We consider the Navier-Stokes equations with Navier's slip boundary conditions in a three-dimensional curved thin domain defined as a thin tubular neighborhood of a given closed surface. Under suitable assumptions we show that the average in the thin direction of a strong solution to the bulk Navier-Stokes equations converges weakly on the limit surface as the thickness of the thin domain tends to zero. Moreover, we derive limit equations on the limit surface by characterizing the limit of the average as a unique weak solution to the limit equations. Our limit equations are the damped Navier-Stokes equations on a closed surface related to the Boussinesq-Scriven surface fluid model, in which the damping term comes from the friction term in the slip boundary conditions of the bulk problem. In particular, if there is no friction between the bulk fluid and the boundary of the curved thin domain, then our limit equations agree with the Navier-Stokes equations on an abstract Riemannian manifold in which the viscous term contains the Ricci curvature as the zeroth order term. This is the first result on the rigorous derivation of the surface Navier-Stokes equations on a general closed surface by the thin-film limit.

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Regularity Estimates for the Stationary Navier-Stokes Equations Over Bumpy Boundaries

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We consider a large-scale Lipschitz estimate for the stationary Navier-Stokes equations above a highly oscillating Lipschitz boundary with the no-slip condition. We will present two methods of proof. The first one comes from the compactness argument in homogenization theory by Avellaneda and Lin (1987, 1989). This method uses a higher-order expansion of solutions involving an oscillating boundary layer corrector, which leads to an improvement of the estimate under a periodicity assumption thanks to the convergence of the boundary layer. The second one grounds on a quantitative excess decay originating from Armstrong and Smart (2016). It does not require any higher-order expansion, and hence, one can study the regularity estimates without knowing the precise behavior of solutions near the boundary. This talk is based on joint work with Christophe Prange (CNRS, Cergy-Pontoise University) and Jinping Zhuge (University of Chicago).

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Vortex Stretching and Enhanced Dissipation for the Incompressible 3D Navier-Stokes Equations

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The zeroth law is the keyword in turbulent energy cascade for which there exist experimental and numerical evidences. For example, in the classical derivation of the Kolmogorov 4/5-law, the zeroth law is significantly assumed (see “Turbulence” written by Frisch). However it seems few studies tried to clarify what kind of vortices formation really induce the zeroth law. From this point of view, we considered the 3D incompressible Navier-Stokes equations under the following 2+1/2-dimensional situation: small-scale horizontal vortex blob (in other word, secondary filament) being stretched by large-scale, anti-parallel pairs of vertical vortex tubes, and then we proved enhanced dissipation (the zeroth law) induced by such vortex-stretching. Note that, there already exist experimental and numerical studies on vortices formation and the energy dissipation. See McKeown, Ostilla-Monico, Pumir, Brenner and Rubinstein (Sci. Adv. 2020). They also observed that the energy dissipation rate increases with the development of the secondary filaments, similar to our mathematical result.

Let us mention a recent numerical simulation which had inspired this mathematical work. Using direct numerical simulations of the 3D Navier-Stokes equations, Goto-Saito-Kawahara (Phys. Rev. Fluids 2017) have found that sustained turbulence consists of a hierarchy of antiparallel pairs of vortex tubes. Their main conclusions can be summarized as follows, which bear some similarity with our constructions of NS solutions:

Turbulence, in the inertial length scales, is composed of hierarchy of vortex tubes with different sizes. At each hierarchy level, vortex tubes tend to form antiparallel pairs and they effectively stretch and create smaller-scale vortex tubes. Moreover, stretched vortex tubes tend to align in the direction perpendicular to larger-scale vortex tubes.

This talk is based on joint works: Jeong-Y (Math. Annal. 2020, Nagare: J. Japan Soc. Fluid Mech. 2020) and Y-Goto-Oka-Tsuruhashi (in preparation).

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Multiphase Flow Analysis of Unsteady Behavior of Cloud Cavitation by the Smoothed Particle Hydrodynamics Method

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Cavitation generates a portion of cavity called a cloud cavitation, which performs collective motions repeating the process of growth and collapse. Especially, it is considered that a high-pressure shock wave is emitted associated with the collapse of the cloud and may cause serious problems such as erosion in fluid machinery. In order to understand such unsteady behaviors of the cloud, much effort has been made mainly for numerical analysis of its internal flow using a simple shape cluster model which is an aggregate of small bubbles. However, it has not been enough understood how the unsteady behaviors of the cloud, namely the inception, growth and collapse are performed from the viewpoint of the multiphase flow. In this study, we make a two-dimensional numerical analysis of the unsteady behavior of the cloud cavitation induced by a submerged bubbly water jet injection into water through a nozzle in order to clarify the mechanism of such behaviors of the cloud in the context of the multiphase flow. For the numerical test, we employ the mixture model of liquids and gases to model the bubbly water jet and utilize the Smoothed Particle Hydrodynamics method to analyze the unsteady flow in Lagrangian description. We examine the numerical test and explain the scenarios of how the cloud is generated in the multiphase flow and of how the unsteady behaviors of the cloud is made. In particular, we show that twin vortices induced by the bubbly water jet play an important role in the inception, growth and shrink processes of the cloud.

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Lagrangian Coherent Structures in Rayleigh-Benard Convection with Perturbations

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Rayleigh-Benard convection is typical natural convection which appears in a fluid layer with heated bottom and cooled top planes. When Rayleigh number Ra is small, the velocity field of the convection is almost steady, but when Ra is gradually increased by raising the temperature difference, perturbations appear in the velocity field. This phenomenon has been modeled as a two-dimensional Hamiltonian model with perturbations and it has been shown that some fluid particles are transported chaotically in the model. However, chaotic transport in Rayleigh-Benard convection has not been studied enough by experimental methods. In our study, we construct an experimental apparatus to measure the two-dimensional velocity field of perturbed Rayleigh-Benard convection by particle image velocimetry (PIV) method and extract invariant structures called Lagrangian coherent structures (LCSs) from the velocity data in order to clarify the global structures of fluid transport. In this talk, we show that LCSs entangle with each other around cell boundaries, creating some lobes as in the Hamiltonian model, and that fluid is transported between cells in accordance with lobe dynamics. We also show in contrast that a figure eight structure, which does not appear in the model, appears in the middle of each cell, making the transport more complicated.

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Dirac Structures and Variational Formulations in Nonequilibrium Thermodynamics

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A Dirac structure is a unifying notion of symplectic and Poisson structures, which has been widely used in mechanics, in particular, for nonholonomic mechanical systems with linear constraints. In this talk, we study Dirac structures appeared in nonequilibrium thermodynamics by extending to a class of nonholonomic systems with nonlinear constraints, together with the associated variational formulation. We illustrate our theory with some examples for the cases of adiabatically closed and open thermodynamic systems. This is a joint work with Francois Gay-Balmaz.

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Gårding Inequalities and Their Impact on Regularity and Uniqueness

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Minimizers of strongly quasiconvex variational integrals need not be regular nor unique. However, if a suitable Gårding type inequality is assumed for the variational integral, then both regularity and uniqueness of minimizers can be restored under natural smallness conditions on the data. In turn, the Gårding inequality turns out to always hold under an a priori C1 regularity hypothesis on the minimizer, while its validity is not known in the general case. In this talk, we discuss these issues and how they are naturally connected to convexity of the variational integral on the underlying Dirichlet classes. We also discuss the problem of regularity for minimizers in the two-dimensional case where stronger results can be established. The talk is based on joint work with Judith Campos Cordero, Bernd Kirchheim and Jan Kolar.

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Asymptotic Properties of Steady Solutions to the 2D Navier-Stokes Equations with the Finite Generalized Dirichlet Integral

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We consider the stationary Navier-Stokes equations in the whole plane $\mathbb{R}^2$ and in the exterior domain outside of the large circle. The solution v is handled in the class with $\nabla v \in L^q$ for $q \geq 2$. Since we deal with the case $q \geq 2$, our class may be larger than that of the finite Dirichlet integrals, i.e., for $q = 2$ where a number of results such as asymptotic behavior of solutions have been observed. For the stationary problem we shall show that $\omega(x) = o(|x|^{-(\frac{1}{q}+\frac{1}{q^2})})$ as $|x| \to \infty$, where $\omega \equiv \operatorname{rot} v$. As an application, we prove the Lioville type theorems under the assumption that $\omega \in L^q(\mathbb{R}^2)$ for $q > 2$.

This talk is based on the joint work with Yutaka Terasawa(Nagoya Univ.) and Yuta Wakasugi(Hiroshima Univ.)

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Friday, 4th December 2020

On the Self-Propelled Motion of a Rigid Body in a Viscous Liquid by Time-Periodic Boundary Data

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We consider a body, $\mathcal{B}$, moving in a Navier-Stokes liquid, $\mathcal{L}$, subject to a driving mechanism constituted by a time-periodic distribution of velocity, $\mathbf{v}_*$, at the interface body-liquid. This study is mostly motivated by the self-propulsion of a "fish," where the net motion is produced by the continuous oscillation of parts of its body. Even though modeling a fish as a rigid body and its moving parts as a boundary velocity distribution might look a bit coarse, it should also be said that, as is well known, the motion of a (deformable) shape-changing object in a liquid can mathematically be reduced –by a suitable transformation– to that of an object of fixed shape with an appropriate distribution of velocity at its boundary. In particular, we show that, in a suitable class of weak solutions, if the average over a period of $\mathbf{v}_*$, $\bar{\mathbf{v}}_*$ is not zero, then $\mathcal{B}$ will propel itself on condition that $\bar{\mathbf{v}}_*$ has a non-vanishing projection on a suitable "control" space. If, on the other hand, $\bar{\mathbf{v}}_* = \mathbf{0}$ (purely oscillatory case), then we show that self-propulsion can occur if and only if $\bar{\mathbf{v}}_*$ satisfies a suitable non-local condition.

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The Time Periodic Problem of the Navier-Stokes Equations in a Bounded Domain with Moving Boundary

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The time periodic problem of the Navier-Stokes equations on a non-cylindrical space-time domain is studied. Time periodicity is one of the fundamental phenomena for incompressible fluid flow and has been studied by many researchers. Here we consider the case in which the domain is changing periodically in time and expect that together with a periodic external force a time periodic flow will occur outside or inside the domain. This problem is called time-periodic problem on a non-cylindrical space-time domain. Motivated by a recent result by J. Saal (2006) on maximal regularity for this kind of system we construct time periodic solutions in L^q-spaces provided the bounded domain moves periodically with small amplitude and the given periodic external force is small. The proof is based on new decay estimates for the solution operator of parabolic evolution equations corresponding to the non-cylindrical space -time domain problem. This is a joint work with Prof. Reinhard Farwig (Technische Universität Darmstadt), Prof. Hideo Kozono (Waseda University) and Dr. David Wegmann.

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On the Two-Phase Navier-Stokes Equations in Unbounded Domains

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In this talk, we consider the two-phase Navier-Stokes equations in unbounded domains. We first introduce the unique solvablility of two-phase elliptic problems related to the Helmholtz decomposition. Then combining this result with a general theory yields the local solvability of the two-phase Navier-Stokes equations. For the rest of the time, we concentrate on the whole space case separated by the sharp interface given by $x_N = h(x', t)$, where $x' = (x_1, \ldots, x_{N-1}) \in \mathbf{R}^{N-1}$ and $N \geq 2$, and discuss time decay properties of solutions for the linearized system.

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Self-Similar Solutions for Compressible Navier-Stokes Equations

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We study the existence of self-similar solutions for compressible Navier-Stokes equations. We will construct two kinds of forward self-similar solutions under the radially symmetric condition, which we can consider with vacuum and without vacuum. We will also discuss backward self-similar solutions.

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