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Regularity results in 2D fluid-structure interaction

Abstract. We study the interaction of an incompressible fluid in two dimensions with an elastic structure yielding the moving boundary of the physical domain. The displacement of the structure is described by a linear viscoelastic beam equation. The main result is the existence of a unique global strong solution. Previously, only the ideal case of a flat reference geometry was considered such that the structure can only move in vertical direction. We allow for a general geometric set-up, were the structure can even occupy the complete boundary.

The main tool is a maximal regularity estimate for the steady Stokes system in domains with minimal boundary regularity. In particular, we can control the velocity in $W^{2,2}$ in terms of a forcing in L^2 provided the boundary belongs roughly to $W^{3/2,2}$. This is applied to the momentum equation in the moving domain (for a fixed time) with the material derivative as right-hand side. Since the moving boundary belongs a priori only to the class $W^{2,2}$, known results do not apply here as they require a C^2-boundary.

Finally we present higher order estimates for the fluid-structure interaction problem. This is based on a parabolic counterpart (in moving domains) of the elliptic estimate for the Stokes system in irregular domains.

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Stability and Turbulence for active fluids in the periodic setting

Abstract. We consider the so called Living Fluid Continuum model which describes the dynamics of living fluids such as bacterial suspensions at low Reynolds number. Generalized Navier-Stokes equations are imposed to model the motion of those active fluids. In this talk we present a full analysis of this model including nonlinear stability analysis in the periodic setting. Thanks to a leading fourth order term (with correct sign) we are able to prove global well-posedness of this system. By making use of Fourier analysis in periodic spaces we are able to prove that a manifold of globally ordered polar states is normally stable resp. normally hyperbolic depending of the choice of the occurring parameters. Here, the principle of linearized (in-)stability is used.

This is a joint work with Christian Gesse and Jürgen Saal.

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On uniformity of the resolvent estimates associated with time-periodic flow past a rotating body

Abstract. We consider the time-periodic viscous flow around a three-dimensional rigid body that rotates and possibly translates along the same axis. Since the linearization of this problem is not well-posed in a setting of classical Sobolev spaces, we introduce a framework of homogeneous Sobolev spaces, where the corresponding resolvent problems are shown to be uniquely solvable. In the case of a pure rotation, one can further derive uniform resolvent estimates, which lead to the existence of solutions to the time-periodic linear proble in spaces of absolutely convergent Fourier series. However, in the case of a rotating and translating body, the uniformity of the resolvent estimates requires additional restrictions, and the existence of time-periodic solutions merely follows if the two present oscillating processes are compatible, that is, if the rotational velocity of the body and the angular velocity associated with the time-periodic forcing are rational multiples of each other. We further consider a counterexample, which suggests that this restriction is even necessary for existence of solutions in the proposed functional framework.

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Solvability of a moving contact-line problem with interface formation for an incompressible viscous fluid

Abstract. A free-boundary problem of a steadily advancing meniscus in a circular capillary tube is investigated. The problem is described using the “interface formation model”, which was originally introduced with the aim of removing the singularities that arise when classical hydrodynamics is applied to problems with a moving contact line. We prove the existence of an axially symmetric solution in weighted Hölder spaces for low meniscus speeds.

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On the asymptotic stability for the two-phase Navier-Stokes equations with a sharp interface

Abstract. In this talk, we consider the motion of two immiscible, incompressible, viscous fluids, *fluid*$_+$ and *fluid*$_-$, in the N-dimensional Euclidean space $\mathbf{R}^N$ for $N \geq 3$. The fluids occupy the regions

$$\Omega_{0\pm} = \{x = (x', x_N) : x' = (x_1, \ldots, x_{N-1}) \in \mathbf{R}^{N-1}, \pm(x_N - \eta_0(x')) > 0\}$$

at the initial time, where η_0 is a given function on $\mathbf{R}^{N-1}$. Here we call *fluid*$_+$ the upper fluid, while *fluid*$_-$ the lower fluid. The motion is governed by the two-phase Navier-Stokes equations with a sharp interface and surface tension is included on the interface. Gravity is also taken into account in our setting. Prüss and Simonett (2010) proved that the Rayleigh-Taylor instability occurs when the upper fluid is heavier than the lower one. On the other hand, we prove the asymptotic stability of the trivial steady state when the lower fluid is heavier than the upper one in this talk. To this end, we first introduce a time-decay estimate of L_p-L_q type for the Stokes semigroup associated with the linearized system of the two-phase Navier-Stokes equations. Combining the time-decay estimate with maximal regularity then shows the asymptotic stability of the trivial steady state for small initial data.

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On magnetoviscoelastic fluids in 3D

Abstract. We show that the system of equations describing a magnetoviscoelastic fluid in three dimensions can be cast as a quasilinear parabolic system. Using the theory of maximal L_p-regularity, we establish existence and uniqueness of local strong solutions and we show that each solution is smooth (in fact analytic) in space and time. Moreover, we give a complete characterization of the set of equilibria and show that solutions that start out close to a constant equilibrium exist globally and converge to a (possibly different) constant equilibrium. Finally, we show that every solution that is eventually bounded in the topology of the state space exists globally and converges to the set of equilibria.

(Joint work with Hengrong Du and Yuanzhen Shao).

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Convergence of a Nonlocal to a Local Diffuse Interface Model for Two-Phase Flow with Unmatched Densities

Abstract. We prove convergence of suitable subsequences of weak solutions of a diffuse interface model for the two-phase flow of incompressible fluids with different densities with a nonlocal Cahn-Hilliard equation to weak solutions of the corresponding system with a standard "local" Cahn-Hilliard equation. The analysis is done in the case of a sufficiently smooth bounded domain with no-slip boundary condition for the velocity and Neumann boundary conditions for the Cahn-Hilliard equation. The proof is based on the corresponding result in the case of a single Cahn-Hilliard equation and compactness arguments used in the proof of existence of weak solutions for the diffuse interface model.

This talk is based on a joint work with Helmut Abels (Regensburg University, Germany).

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Free Boundary Problems via Da Prato - Grisvard Theory

Abstract. A common way to prove global well-posedness of free boundary problems for incompressible viscous fluids is to transform the equations governing the fluid motion to a fixed domain with respect to the time variable. An elegant and physically reasonable way to do this is to introduce Lagrangian coordinates. These coordinates are given by the transformation rule

$$x(t) = \xi + \int_0^t u(\tau, \xi) \, \mathrm{d}\tau,$$

where $u(\tau, \xi)$ is the velocity vector of the fluid particle at time τ that initially started at position ξ. The variable $x(t)$ is then the so-called Eulerian variable and belongs to the coordinate frame where the domain that is occupied by the fluid moves with time. The variable ξ is the Lagrangian variable that belongs to time fixed variables. In these coordinates the fluid only occupies the domain Ω_0 that is occupied at initial time $t = 0$. To prove a global existence result for such a problem,

it is important to guarantee the invertibility of this coordinate transform globally in time. By virtue of the inverse function theorem, this is the case if

$$\nabla_\xi x(t) = \mathrm{Id} + \int_0^t \nabla_\xi u(\tau, \xi)\, \mathrm{d}\tau$$

is invertible. By using a Neumann series argument, this is invertible, if the integral term on the right-hand side is small in $\mathrm{L}^\infty(\Omega_0)$. Thus, it is important to have a global control of this L^1-time integral with values in $\mathrm{L}^\infty(\Omega_0)$. If the domain is bounded, this can be controlled by decay properties of the corresponding semigroup operators that describe the motion of the linearized fluid equation. On certain unbounded domains, however, these decay properties are not true anymore. While there are technical possibilities to fix these problems if the boundary is compact, these fixes cease to work if the boundary is non-compact.

As a model problem, we consider the case where Ω_0 is the upper half-space. To obtain estimates of the L^1-time integral we establish a homogeneous version of the celebrated theorem of Da Prato and Grisvard (1975) about maximal regularity in real interpolation spaces. In these lectures, we will describe this homogeneous Da Prato–Grisvard theorem in detail and show how it can be applied to solve problems from fluid mechanics involving a free non-compact boundary.

This is a joint work with Raphaël Danchin, Matthias Hieber, and Piotr Mucha.

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Global wellposedness of 3D compressible Navier-Stokes equations with free surface in Lagrangian approach

Abstract. In this talk, we discuss about the problem concerning the motion of the barotropic compressible Navier-Stokes equations (CNS) in the smooth exterior domain with some free surface. By applying the method of (full or partial) Lagrangian coordinates, the nonlinear problem reduces to the study of the Lame system with the free boundary conditions. Then the L_p-L_q decay estimates are established for such linearized system, by taking advantage of the local energy approach. At last, we apply the L_p-L_q decay theory to construct the global solution of (CNS). Using the full Lagrangian coordinates, we can find the global unique solution in some maximal regularity class. On the other hand, we also establish the classical Matsumura-Nishida theory for the free boundary value problem via the partial Lagrangian approach.

All the results above are based on the joint works with Prof. Yoshihiro Shibata from Waseda University.