Hideo KOZONO

Abstract of research

Characterization of the initial data via homogeneous Besov space for the existence of strong solutions of the Navier-Stokes equations

Consider the Cauchy problem of the Navier-Stokes equations in $\mathbb{R}^n$ with initial data a in the homogeneous Besov space $\dot{B}^{-1+\frac{n}{p}}_{p,q}(\mathbb{R}^n)$ for $n < p < \infty$ and $1 \leqq q \leqq \infty$. We show that the Stokes flow $e^{t\Delta}a$ can be controlled in $L^{\alpha,q}(0,\infty;\dot{B}^0_{r,1}(\mathbb{R}^n))$ for $\frac{2}{\alpha} + \frac{n}{r} = 1$ with $p \leqq r \leqq \infty$, where $L^{\alpha,q}$ denotes the Lorentz space. As an application, the global existence theorem of mild solutions for the small initial data is established in the above class which is slightly stronger than Serrin's. Conversely, if the global solution belongs to the usual Serrin class $L^{\alpha,q}(0,\infty;L^r(\mathbb{R}^n))$ for r and α as above with $1 < q \leqq \infty$, then the initial data a necessarily belongs to $\dot{B}^{-1+\frac{n}{r}}_{r,q}(\mathbb{R}^n)$. Moreover, we prove that such solutions are analytic in the space variables. Our method for the proof of analyticity is based on a priori estimates of higher derivatives of solutions in $L^p(\mathbb{R}^n)$ with Hölder continuity in time $(0,\infty)$.

Characterization of L^r-harmonic vector fields in 2D exterior domains

Consider the space of harmonic vector fields h in $L^r(\Omega)$ for $1 < r < \infty$ for the *two* dimensional *exterior* domain Ω with the smooth boundary $\partial\Omega$ subject to the boundary conditions $h \cdot \nu = 0$ or $h \wedge \nu = 0$, where ν denotes the unit outward normal to $\partial\Omega$. Denoting these spaces by $X^r_{\mathrm{har}}(\Omega)$ and $V^r_{\mathrm{har}}(\Omega)$, respectively, it is shown that, in spite of the lack of compactness of Ω, both of these spaces are finite dimensional and that their dimension of both spaces coincides with L for $2 < r < \infty$ and $L-1$ for $1 < r \leq 2$. Here L is the number of disjoint simple closed curves consisting of the boundary $\partial\Omega$.

Publications

1. Kozono, H., Okada, A., Shimizu, S., Characterization of initial data in the homogeneous Besov space for solutions in the Serrin class of the Navier-Stokes equations J. Funct. Anal. **278** (2020), 108390. https://doi.org/10.1016/j.jfa.2019.108390
2. Hieber M., Kozono, H., Seyfert, A., Shimizu.,S., Yanagaisawa,T., A Characterization of Harmonic Lr-Vector Fields in Two-Dimensional Exterior Domains. J. Geom. Anal. **30** (2020), 3742–3759. https://doi.org/10.1007/s12220-019-00216-0

Talks

1. International Workshop on Multiphase Flows: Analysis, Modelling and Numerics 1-4 December 2020, Waseda University, Tokyo, Japan December 3, 2020.

   Title : **Asymptotic properties of steady solutions to the 2D Navier-Stokes equations with the finite generalized Dirichlet integral.**

   We consider the stationary Navier-Stokes equations in the whole plane $\mathbb{R}^2$ and in the exterior domain outside of the large circle. The solution v is handled in the class with

$\nabla v \in L^q$ for $q \geq 2$. Since we deal with the case $q \geq 2$, our class may be larger than that of the finite Dirichlet integrals, i.e., for $q = 2$ where a number of results such as asymptotic behavior of solutions have been observed. For the stationary problem we shall show that $\omega(x) = o(|x|^{-(\frac{1}{q}+\frac{1}{q^2})})$ as $|x| \to \infty$, where $\omega \equiv \mathrm{rot}\ v$. As an application, we prove the Lioville type theorems under the assumption that $\omega \in L^q(\mathbb{R}^2)$ for $q > 2$. This talk is based on the joint work with Yutaka Terasawa(Nagoya Univ.) and Yuta Wakasugi(Hiroshima Univ.)

2. Series of lectures at Beijing University December 4, 11, 18 and 25, 2020.

   Title : **Strong solutions of the Navier-Stokes equations based on the maximal Lorentz regularity theorem in Besov spaces (I), (II), (III), (IV), (V).**

   We show existence and uniqueness theorem of local strong solutions to the Navier-Stokes equations with arbitrary initial data and external forces in the homogeneous Besov space with both negative and positive differential orders which is an invariant space under the change of scaling. If the initial data and external forces are small, then the local solutions can be extended globally in time. Our solutions also belong to the Serrin class in the usual Lebesgue space. The method is based on the maximal Lorentz regularity theorem of the Stokes equations in the homogeneous Besov spaces. As an application, we may handle such singular data as the Dirac measure and the single layer potential supported on the sphere. This is the joint work with Prof. Senjo Shimizu at Kyoto University.

   https://www.math.pku.edu.cn/kxyj/xsbg/tlb/geometricanalysis/124620.htm

3. Fudan International Seminar on Analysis, PDEs, and Fluid Mechanics March 11, 2021.

   Title: **L^r-Helmholtz-Weyl decomposition in two dimensional exterior domains.**

   In 2D exterior domains, we consider two spaces of $L-r$-harmonic vector fields which are parallel or perpendicular to the boundary. We first show that these spaces are of finite dimensions for all $1 < r < \infty$. In particular, if the domain has L-connected components of the boundary, then the dimension is characterized by L with its threshold $r = 2$. We next show that every L^r-vector fields is decomposed by the sum of harmonic vector fields, rotation and gradient of scalar functions for all $1 < r < \infty$. However, uniqueness of such a decomposition holds only for $1 < r \leq 2$. This talk is based on the joint work with Matthias Hieber, Anton Seyfert(TU Darmstadt), Senjo Shimizu(Kyoto Univ.) and Taku Yanagisawa(Nara Women Univ.).