Hideo KOZONO

Abstract of research

Strong solutions of the Navier-Stokes equations based on the maximal Lorentz regularity theorem in Besov spaces

We show existence and uniqueness theorem of local strong solutions to the Navier-Stokes equations with arbitrary initial data and external forces in the homogeneous Besov space with both negative and positive differential orders which is an invariant space under the change of scaling. If the initial data and external forces are small, then the local solutions can be extended globally in time. Our solutions also belong to the Serrin class in the usual Lebesgue space. The method is based on the maximal Lorentz regularity theorem of the Stokes equations in the homogeneous Besov spaces. As an application, we may handle such singular data as the Dirac measure and the single layer potential supported on the sphere.

Maximal regularity of the Stokes operator in an exterior domain with moving boundary and applications to the Navier-Stokes equations

Consider the Navier-Stokes system on an exterior domain with moving boundary and Dirichlet boundary conditions. In 2003 J. Saal proved that the Stokes operator in a domain with moving boundary has the property of maximal regularity provided that the operator is invertible. Hence his result can be applied if the domain is bounded or by adding a shift to the Stokes operator if the domain is unbounded or the time interval is finite. In this paper, we will generalize his result to a result global in time if the reference domain is an exterior domain. Finally, we will apply this result to the Navier-Stokes equations to obtain a global in time existence theorem for small data.

Stationary solution to the Navier-Stokes equations in the scaling invariant Besov space and its regularity We consider the stationary problem of the Navier-Stokes equations in $\mathbb{R}^n$ for $n \geq 2$. We show existence, uniqueness and regularity of solutions in the homogeneous Besov space $\dot{B}_{p,q}^{-1+\frac{n}{p}}(\mathbb{R}^n)$ which is the scaling invariant one. As a corollary of our results, a self-similar solution is obtained. For the proof, several bilinear estimates are established. The essential tool is based on the paraproduct formula and the imbedding theorem in homogeneous Besov spaces. In particular, our space enables us to obtain a general existence theorem of solutions in $\mathbb{R}^2$, while the usual Lebesgue space $L^2(\mathbb{R}^2)$ is unavailable due to lack of the bilinear estimate associated with the nonlinear structure .

Time global existence and finite time blow-up criterion for solutions to the Keller-Segel system coupled with the Navier-Stokes fluid

We deal with the chemotaxis model under the effect of the Navier-Stokes fluid, *i.e.*, the incompressible viscous fluid. We show the existence of a local *mild solution* for large initial data and a global *mild solution* for small initial data in the scale invariant class demonstrating that $n_0 \in L^1(\mathbb{R}^2)$ and $u_0 \in L^2_\sigma(\mathbb{R}^2)$. Our method is based on the perturbation of linearization together with the L^p-L^q-estimates of the heat semigroup. As a by-product of our method, we prove the smoothing effect and uniqueness of our *mild solution.* In addition, we show a blow-up criterion which almost covers the well-known threshold number 8π of the size $\|n_0\|_{L^1(\mathbb{R}^2)}$ under the rest state of the fluid motion. Furthermore, the blow-up rate isalso discussed.

Publications

1. Kozono, H., Shimizu, S., Strong solutions of the Navier-Stokes equations based on the maximal Lorentz regularity theorem in Besov spaces. J. Funct. Anal. **276** (2019), 896–931. https://doi.org/10.1016/j.jfa.2018.06.006
2. Farwig, R., Kozono, H., Wegmann,D., Maximal regularity of the Stokes operator in an exterior domain with moving boundary and application to the Navier-Stokes equations. Math. Ann. **375** (2019), 949–972. https://doi.org/10.1007/s00208-018-1773-x
3. Kaneko,K., Kozono, H., Shimizu, S., Stationary solution to the Navier-Stokes equations in the scaling invariant Besov space and its regularity. Indiana Univ. Math. J. **68** (2019), 857–880.http://dx.doi.org/10.1512/iumj.2019.68.7650
4. Kozono, H., Miura, M., Sugiyama, Y., Time global existence and finite time blow-up criterion for solutions to the Keller-Segel system coupled with the Navier-Stokes fluid. J. Differential Equations **267** (2019), 5410–5492. https://doi.org/10.1016/j.jde.2019.05.035

Talks

1. International Conference Nonlinear Analysis, Palazzone, Scuola Normale Superiore di Pisa, Cortona, Italy 11–14 June 2019, June 12, 2019

   Title : **Asymptotic properties of steady solutions to the 2D Navier-Stokes equations with the finite generalized Dirichlet integral.**

   We consider the stationary Navier-Stokes equations in the whole plane $\mathbb{R}^2$ and in the exterior domain outside of the large circle. The solution v is handled in the class with $\nabla v \in L^q$ for $q \geq 2$. Since we deal with the case $q \geq 2$, our class may be larger than that of the finite Dirichlet integrals, i.e., for $q = 2$ where a number of results such as asymptotic behavior of solutions have been observed. For the stationary problem we shall show that $\omega(x) = o(|x|^{-(\frac{1}{q}+\frac{1}{q^2})})$ as $|x| \to \infty$, where $\omega \equiv \mathrm{rot}\ v$. As an application, we prove the Lioville type theorems under the assumption that $\omega \in L^q(\mathbb{R}^2)$ for $q > 2$.
2. RIMS「Mathematical Fluid Mechanics」, Kyoto University July 3–5, July 4, 2019

   Title : **L^r-Helmholtz-Weyl decomposition in 3D exterior domains.**
3. Kagurasaka Analysis Seminar , Tokyo University of Science, July 27, 2019

   Title : **L^r-Helmholtz-Weyl decomposition in 3D exterior domains.**
4. XI Workshop on Nonlinear Differential Equations, Varese, Italy, July 29th - August 2nd, 2019, July 30, 2019

   Title : **Asymptotic properties of the 2D Navier-Stokes flows.**
5. Tohoku University, Seminar on Geometry October 5, 2019

   Title : **L^r-Helmholtz-Weyl decomposition of vector fields in 3D exterior domains.**

6. The 26th Colloquium at Department of Mathematics, Waseda University October 24, 2019.

   Title : **L^r-Helmholtz-Weyl decomposition in 3D exterior domains.**

7. Evolution Equations: Abstract and Applied Perspectives in Honour of the 60th Birthday of Matthias Hieber, Luminy, CIRM, France, October 28th–November 1st, 2019, October 28, 2019

   Title : **L^r-Helmholtz-Weyl decomposition in 3D exterior domains.**

   It is known that in 3D exterior domains Ω with the compact smooth boundary $\partial\Omega$, two spaces $X^r_{\mathrm{har}}(\Omega)$ and $V^r_{\mathrm{har}}(\Omega)$ of L^r-harmonic vector fields $\boldsymbol{h}$ with $\boldsymbol{h} \cdot \boldsymbol{\nu}|_{\partial\Omega} = 0$ and $\boldsymbol{h} \times \boldsymbol{\nu}|_{\partial\Omega} = 0$ are both of finite dimensions, where $\boldsymbol{\nu}$ denotes the unit outward normal to $\partial\Omega$. We prove that for every L^r-vector field $\boldsymbol{u}$, there exist $\boldsymbol{h} \in X^r_{\mathrm{har}}(\Omega)$, $\boldsymbol{w} \in \dot{H}^{1,r}(\Omega)^3$ with div $\boldsymbol{w} = 0$ and $p \in \dot{H}^{1,r}(\Omega)$ such that $\boldsymbol{u}$ is uniquely decomposed as

   $$\boldsymbol{u} = \boldsymbol{h} + \mathrm{rot}\ \boldsymbol{w} + \nabla p.$$

   On the other hand, if for the given L^r-vector field $\boldsymbol{u}$ we choose its harmonic part $\boldsymbol{h}$ from $V^r_{\mathrm{har}}(\Omega)$, then we have a similar decomposition to above, while the unique expression of $\boldsymbol{u}$ holds only for $1 < r < 3$. Furthermore, the choice of p in $\dot{H}^{1,r}(\Omega)$ is determined in accordance with the threshold $r = 3/2$.

8. Mathematical Socity of Japan・Branch of Functional Equations 「Bibunhoteishiki no Sougouteki Kenkyu」 December 21–22, 2019 at Tokyo Institute of Technology, December 21–22, 2019

   Title : **Survey on the L^r-Helmholtz-Weyl decomposition in 3D exterior domains**

9. Colloquium at the Institute of Polymathematics, Nagoya University. January 15, 2020

   Title : **L^r-harmonic vector fields and Helmholtz-Weyl decomposition in 3D exterior domains**

   We consider the de Rham-Hodge-Kodaira decomposition of L^r-vector fields in 3-dimensional exterior domains with the smooth boundary. The boundary conditions which we handle are two types, that is, tangential and perpendicular to the boundary. First, we shall show that harmonic vector fields in L^r with these boundary conditions are both of finite dimension. It is clarified that in contrast to the case of bounded domains, the number of dimension depends on the integral exponent r. Even in exterior domains, we define the Betti number L which is topologically invariant, and characterize the dimension by means of the r and L. Next, we shall show that every L^r-vector fields can be decomposed into the sum of harmonic vector field, the range of rotation and gradient of vector and scalar potentials, respectively. The unique expression of the sum is closely related to the boundary condition of the harmonic vector field and the integral exponent with the threshold $r = 3$. Our result is based on the joint work with Prof.Matthias Hieber, Dr.Anton Seyfert(TU Darmstadt), Prof.Senjo Shimizu(Kyoto Univ.) and Prof.Taku Yanagisawa(Nara Women Univ.).

10. Kickoff meeting on collaboration between Tohoku University and Tokyo University of Science, at the Kagurazaka Campus of Tokyo University of Science March 4, 2020

Title : **L^r-Helmholtz-Weyl decomposition in 2D and 3D exterior domains**