

## 2017—2018 Research content

I established a methodology to show the maximal regularity theorem for the initial value problem with non-homogeneous boundary condition of the system of parabolic equations by constructing  $R$  bounded solution operator of the corresponding generalized resolvent problems. This theory is an unprecedented new method based on the theory of harmonic analysis on functions taking valued in infinite dimensional space established in the early 21<sup>st</sup> century. The maximal regularity theorem for the initial value problem with homogeneous boundary conditions has been established based on the semigroup theory, but the non-homogeneous boundary condition case could not be handled. One can deal with the case of non-homogeneous boundary conditions by the method using  $R$  bounded solution operator, and its application enable us to build with the theory of the local in time unique existence of solutions for quasi-linear systems of parabolic equations in the function space with different exponents of time  $L_p$  and space  $L_q$ . As a result, the unique solvability theory of the free boundary value problem of the Navier-Stokes equation is developed dramatically. I also constructed a unique solvability theory in the maximal regularity space, which is the time  $L_p$  and space  $L_q$ , for the system of parabolic equations such as the viscoelastic model and the Q-tensor model for liquid crystals.

## 2019 research project

- 1) It turns out that the  $R$  bounded solution operator can be applied to the maximal regularity theory of the high frequency part of the periodic solution of the system of parabolic equations with inhomogeneous boundary conditions, and so I will construct this detailed theory and it will be applied to show the existence of periodic solutions of the quasi-linear system of parabolic equations. This will be a joint research with Mads Kyed and Thomas Eiter in Germany.
- 2) Perform mathematical analysis of droplet drop problem due to external force such as gravity. This is also a joint research with Mads Kyed and Thomas Eiter.
- 3) To investigate the unique existence of local in time and global in time solutions of the system of MHD equations with transmission conditions.
- 4) Two-phase problems of compressible viscous fluid and compressible—incompressible viscous fluids are studied separately with and without surface tension.

## List of papers published in 2017 and 2018

[1] Robert Denk and Yoshihiro Shibata,

Generation of semigroups for the thermoelastic plate equation with free boundary condition,

Evolution Equation and Control Theory (EECT)

Vol 8 (2) (2019), 301--313. doi:10.3934/eect2019016

[2] Maria Schonbeck and Yoshihiro Shibata,  
Global well-posedness and decay for a Q tensor model of Incompressible Nematic  
Liquid Crystals in  $\mathbb{R}^N$ ,

J. Differential Equations Vol 266 (2019) 3034--3065

[3] Yoshihiro Shibata

Global well-posedness of unsteady motion of viscous incompressible capillary liquid  
bounded by a free surface}, Evolution Equations and Control Theory,

Vol 7 (1) (2018) 117--152, doi:10.3934/eect.2018007

[4] Yoshihiro Shibata,

On the  $L_p$ - $L_q$  decay estimate for the Stokes equations with free boundary conditions in  
an exterior domain}, Asymptotic Analysis

Vol 107(1-2) (2018), 33--72, DOI: 10.3233/ASY-171449

[5] Yoshihiro Shibata,

On the local wellposedness of free boundary problem for the Navier-Stokes equations in  
an exterior domain,

Communication on Pure and Applied Analysis Vol 17 (4) July (2018),1681--1721,

DOI: 10.3934/cpaa.2018081

[9] Yoshihiro Shibata and Suma'Inna,

On the maximal  $L_p$ - $L_q$  theory arising in the study of a free boundary problem for the  
Navier-Stokes equations}, eBooks MedCrave Group LLC, Sept. 17, 2018

<https://medcraveebooks.com/ebooks/view%eBook/76>

[6] Maria Schonbeck and Yoshihiro Shibata,

On a global well-posedness of strong dynamics of incompressible nematic liquid  
crystals in  $\mathbb{R}^N$ ,

J. Evol. Equ. Vol 17(1) (2017), 537--550. doi:10.1007/s00028-016-0358-y

[7] Yoshihiro Shibata,

Global wellposedness of a free boundary problem for the Navier-Stokes equations in an  
exterior domain,

Fluid Mech Res Int, Vol 1 (2) (2017),

DOI:10.15406/fimrij.2017.01.00008

[8] Yuko Enomoto and Yoshihiro Shibata,

Global Existence of Classical Solutions and Optimal Decay Rate Via the Theory of  
Semigroup,

Y. Giga and A. Novotny (eds.) Handbook of Mathematical

Analysis in Mechanics of Viscous Fluids, Springer International

Pub. AG 2017. DOI [10.1007/978-3-319-10151-4\\_42-1](https://doi.org/10.1007/978-3-319-10151-4_42-1)

[10] Robert Denk and Yoshihiro Shibata,

Maximal regularity for the thermoelastic plate equations with free boundary conditions,

J. Evol. Equ. Vol 17(1) (2017), 215-261.

doi:[10.1007/s00028-016-0367-x](https://doi.org/10.1007/s00028-016-0367-x)

[11] Maria Schonbeck and Yoshihiro Shibata,

On a global well-posedness of strong dynamics of incompressible nematic liquid  
crystals  $\mathbb{R}^N$

J. Evol. Equ. Vol 17(1) (2017), 537--550.

doi:[10.1007/s00028-016-0358-y](https://doi.org/10.1007/s00028-016-0358-y)